Understanding Attribute Acceptance Sampling

Dan O'Leary CBA, CQA, CQE, CRE, SSBB, CIRM
President
Ombu Enterprises, LLC
Dan@OmbuEnterprises.com
www.OmbuEnterprises.com
603-358-3082

OMBU
ENTERPRISES, LLC

Outline

- Sampling Plan Concepts
- ANSI/ASQ Z1.4
- Single Sampling Plans
- Double and Multiple Sampling Plans
- c=0 Sampling Plans
- Summary and Conclusions
- Questions

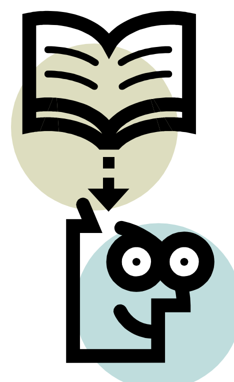
Sampling Plans

Some Initial Concepts

A Typical Application

- You receive a shipment of 5,000 widgets from a new supplier.

- Is the shipment good enough to put into your inventory?



How will you decide?

You Have a Few Approaches

- Consider three potential solutions
  - Look at all 5,000 widgets (100% inspection)
  - Don’t look at any, put the whole shipment into stock (0% inspection)
  - Look at some of them, and if enough of those are good, keep the lot (Acceptance sampling)

- In a sampling plan, we need to know:
  - How many to inspect or test?
  - How to distinguish “good” from “bad”?
  - How many “good” ones are enough?

Two Kinds of Information

Attributes

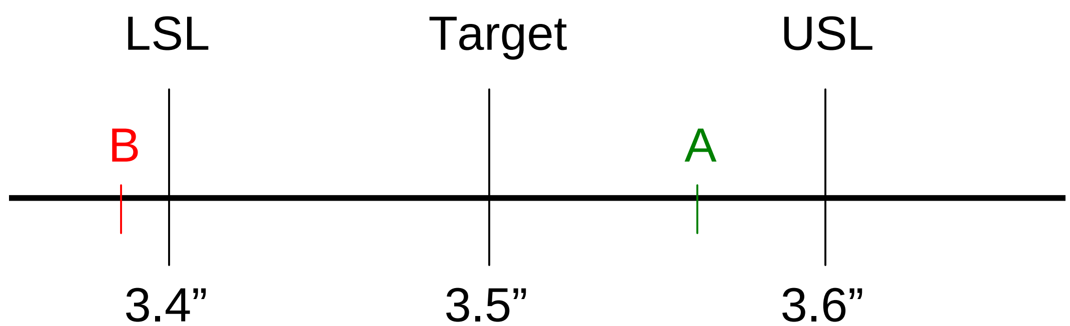
- We classify things using attributes
  - A stop light can be one of three colors: red, yellow, or green
  - The weather can be sunny, cloudy, raining, or snowing
  - A part can be conforming or nonconforming

Variables

- We measure things using variables
  - The temperature of the oven is 350° F
  - The tire pressure is 37 pounds per square inch (psi).
  - The critical dimension for this part number is 3.47 inches.

Convert Variables To Attributes

- Consider an important dimension with a specification of 3.5±0.1 inches.
  - Piece A, at 3.56 inches is conforming.
  - Piece B, at 3.39 inches is nonconforming.



Specification is 3.5±0.1

A Note About Language

- Avoid “defect” or “defective”
  - They are technical terms in the quality profession, with specific meaning
  - They are also technical terms in product liability, with a different meaning
  - They have colloquial meaning in ordinary language

- I encourage the use of “nonconformances” or “nonconforming”

Two Attribute Sampling Plans

- ANSI/ASQ Z1.4 *Sampling Procedures and Tables for Inspection By Attributes*

- ISO 2859-1 *Sampling procedures for inspection by attributes – Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

- ANSI/ASQ Z1.4 and ISO 2859-1 are the classical methods evolved from MIL-STD-105

- The c=0 plans are described in *Zero Acceptance Number Sampling Plans* by Squeglia

Acceptance Sampling is Common . . .

- The most common place for acceptance sampling is incoming material
  - A supplier provides a shipment, and we judge its quality level before we put it into stock.

- Acceptance sampling (with rectifying inspection) can help protect from processes that are not capable

- Destructive testing is also a common application of sampling

. . . Acceptance sampling isn't always appropriate

- Acceptance sampling is **not** process control
- Statistical process control (SPC) is the preferred method to prevent nonconformances.
- Think of SPC as the control method, and acceptance sampling as insurance
- You practice good driving techniques, but you don't cancel your insurance policy

Attribute Sampling Plans

Single Sample Example

We start with an exercise, and then explain how it works

- Your supplier submits a lot of 150 widgets and you subject it to acceptance sampling by attributes.
- The inspection plan is to select 20 widgets at random.
  - If 2 or fewer are nonconforming, then accept the shipment.
  - If 3 or more are nonconforming, then reject the shipment.

In symbols:
N =150
n = 20
c = 2, r = 3

This is a Z1.4 plan that we will examine.

Here is the basic approach

- Select a single simple random sample of $n = 20$ widgets.
- Classify each widget in the sample as conforming or nonconforming (attribute)
- Count the number of nonconforming widgets
- Make a decision (accept or reject) on the shipment
- Record the result (quality record)

Attribute Sampling Plans

ANSI/ASQ Z1.4

Current status of the standards

- MIL-STD-105
  - The most recently published version is MIL-STD-105E
  - Notice 1 cancelled the standard and refers DoD users to ANSI/ASQC Z1.4-1993

- ANSI/ASQ Z1.4
  - Current version is ANSI/ASQ Z1.4: 2008

- FDA Recognition
  - The FDA recognizes ANSI/ASQ Z1.4-2008 as a General consensus standard

Extent of Recognition: Use of all Single, Double and Multiple sampling plans according to the standard's switching rules to make acceptance/rejection decisions on a continuous stream of lots for a specified Acceptance Quality Limit (AQL).

Getting started with Z1.4

- To correctly use Z1.4, you need to know 5 things
  - Lot Size
  - Inspection Level
  - Single, Double, or Multiple Sampling
  - Lot acceptance history
  - AQL

The Flow of Information

We develop our example in these boxes, by showing the choice we make.

Lot Size

Inspection Level

Code Letter (Tbl. I)

S/D/M

Table II, III, or IV

N/R/T

Sub-table A, B, or C

AQL

Sampling Plan n_i, c_i, & r_i

Traditional Information Sources
Purchasing – Lot Size
Quality Engineer – Inspection Level, S/D/M, AQL
Lot History – N/R/T

Lot Size

- The lot size is the number of items received at one time from the supplier.

Received 150 pieces
N = 150

- For incoming inspection, think of it as the quantity on the pack slip.
- The Purchase Order (or contract) typically sets the lot size.

Inspection Level

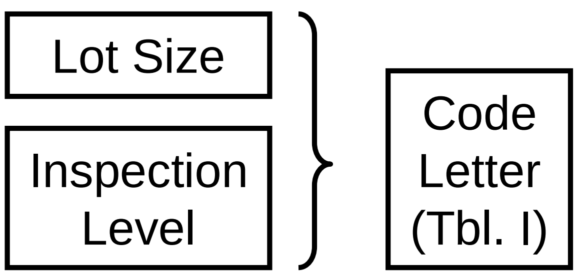
- The inspection level determines how the lot size and the sample size are related
  - Z1.4 provides seven different levels: S1, S2, S3, S4, I, II, and III.
  - Use Inspection Level II unless you have a compelling reason to do something else.

Use Level II

- The Quality Engineer sets the Inspection Level.

Code Letter

- The Inspection Level and Lot Size combine to determine the code letter.
  - Use Table I to determine the code letter.



Level II
Lot Size = 150
Code Letter = F

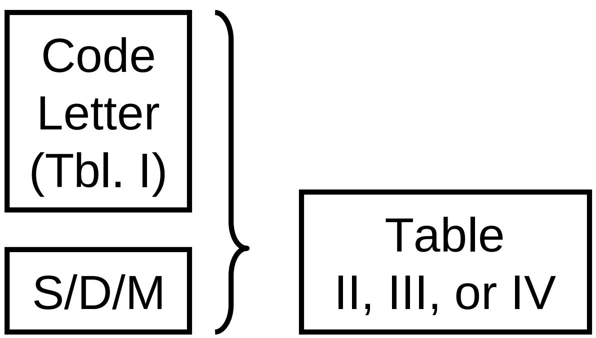
Single, Double, or Multiple Sampling (S/D/M)

- Decide the type of sampling plan (Single, Double, or Multiple)

- This is a balance between average sample number (ASN) and administrative difficulty.

- Generally, moving from single to double to multiple
  - The ASN goes down
  - The administrative difficulty goes up

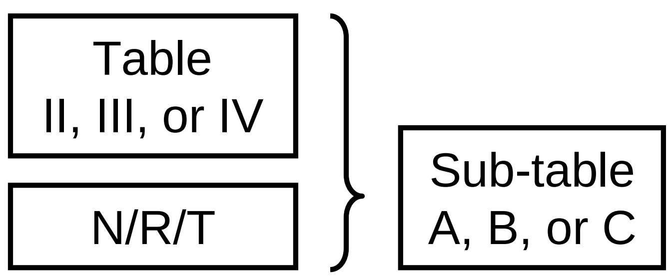
Use Single Sampling Table II



Lot acceptance history

- Z1.4 uses a system of switching rules

- Based on the lot history, we inspect the same (normal), less (reduced), or more (tightened).



Inspection States

- The system can be in one of four states:
  - Normal
  - Reduced
  - Tightened or
  - Discontinue

Use Normal

AQL

- We will discuss AQL shortly
  - Z1.4 uses the AQL to index the sampling plans.
  - The supplier's process average should be as low as possible, but certainly less than the Z1.4 AQL.

Use AQL of 4.0

- The Quality Engineer sets the AQL.

Sampling Plan

- The type and history get us to the right table.
- The Code Letter and AQL get us to the sampling plan.
- Note, however, that you may have to use the “sliders” to get the sampling plan.

Sample Size = 20
Accept on 2
Reject on 3

Sub-table A, B, or C

AQL

Sampling Plan n_i, c_i, & r_i

The Sliders

- Sometimes the Code Letter, Level, and AQL don't have a plan.
  - Z1.4 will send you a different plan using the "sliders" These are arrows pointing up or down.
  - Use the new plan (with the new code letter, sample size, accept number, and reject number).
  - Example: Code letter D & AQL=4.0 doesn't have a plan
    - "Slide" down to N=13, Ac=1, Re = 2

Changing the lot size

- You supplier has been shipping 150 units in the lot, based on the Purchase Order, for a long time.
- Your supplier calls your buyer and says, “We were near the end of a raw material run, and made 160 widgets, instead of 150. Can I ship all 160 this time?”
- The buyer says, “Sure no problem. I’ll send a PO amendment.”
- What is the sampling plan?
  - Answer: n = 32, c = 3, r = 4

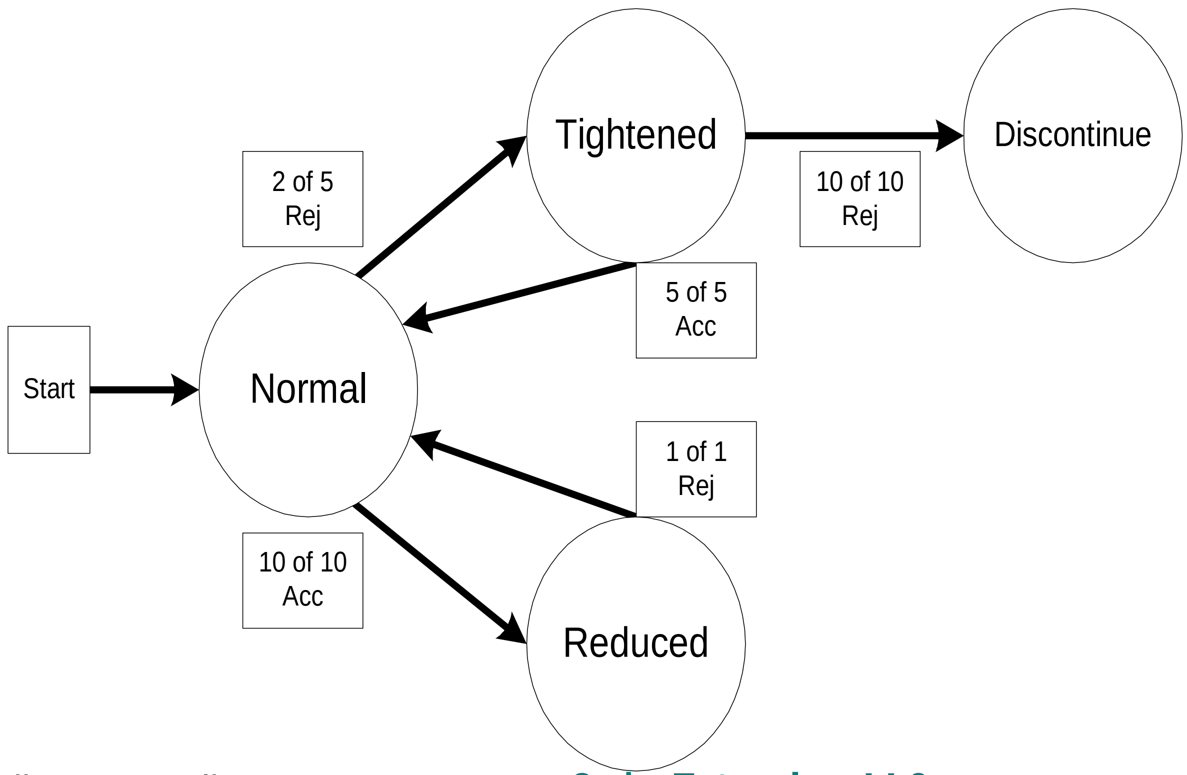
The Code Letter becomes G

Sample Size = 32
Accept on 3
Reject on 4

Sampling Schemes

- Z1.4 tracks the history of lot acceptance and the sampling plans as a result.
  - Consistently good history can reduce the sample size
  - Consistently poor history can shift the OC Curve

- The figure is a simplified version of the switching rules



An Example of Switching Rules

- Look at the handout called “Attribute Sampling Log”
- This is a hypothetical example of how to keep track of the inspection history
- We will follow the history to see an application of the switching rules

Sampling

Some Common Concepts

Sampling With/Without Replacement

- When we took the widget sample, we didn't put them back into the lot during sampling, *i.e.,* we didn't replace them.

- This changes the probabilities of the rest of the lot.
  - If the lot is large, it doesn't make too much difference.
  - For small lots we need the hypergeometric distribution for the calculation.

- In acceptance sampling we sample without replacement!

Simple *v*. Stratified Sampling

- Assume the lot has *N* items
  - In a <u>simple random sample</u> each piece in the lot has equal probability of being in the sample.
  - In a <u>stratified sample</u>, the lot is divided into *H* groups, called strata. Each item in the lot is in one and only one stratum.

- You receive a shipment of 5,000 AAA batteries in 50 boxes of 100 each.
  - First you take a sample of the boxes, then you take a sample of the batteries in the sampled boxes
  - This is a stratified sample: $N=5{,}000$ & $H=50$.

Our Conventions

- Unless we say otherwise we make the following conventions
  - Sampling is performed without replacement
  - Sampling is a simple random sample

The Binomial Distribution

Bernoulli Trial

- Bernoulli trials are a <u>sequence</u> of *n* <u>independent</u> trials, where each trial has only two possible outcomes.

- Example – Flip a coin fifty times
  - This is a sequence of trials
  - $n = 50$
  - The trials are independent, because the coin doesn't "remember" the previous trial
  - The only outcome of each trial is a head or a tail

The Binomial Distribution

- The Bernoulli trial has two possible outcomes.
  - One outcome is "success" with probability *p*.
  - The other "failure" with probability $q = 1 - p$.

- The binomial distribution is the probability of *x* successes in *n* trials

$$\Pr(x) = \binom{n}{x} p^x (1-p)^{n-x},\ x = 0, 1, \cdots, n$$

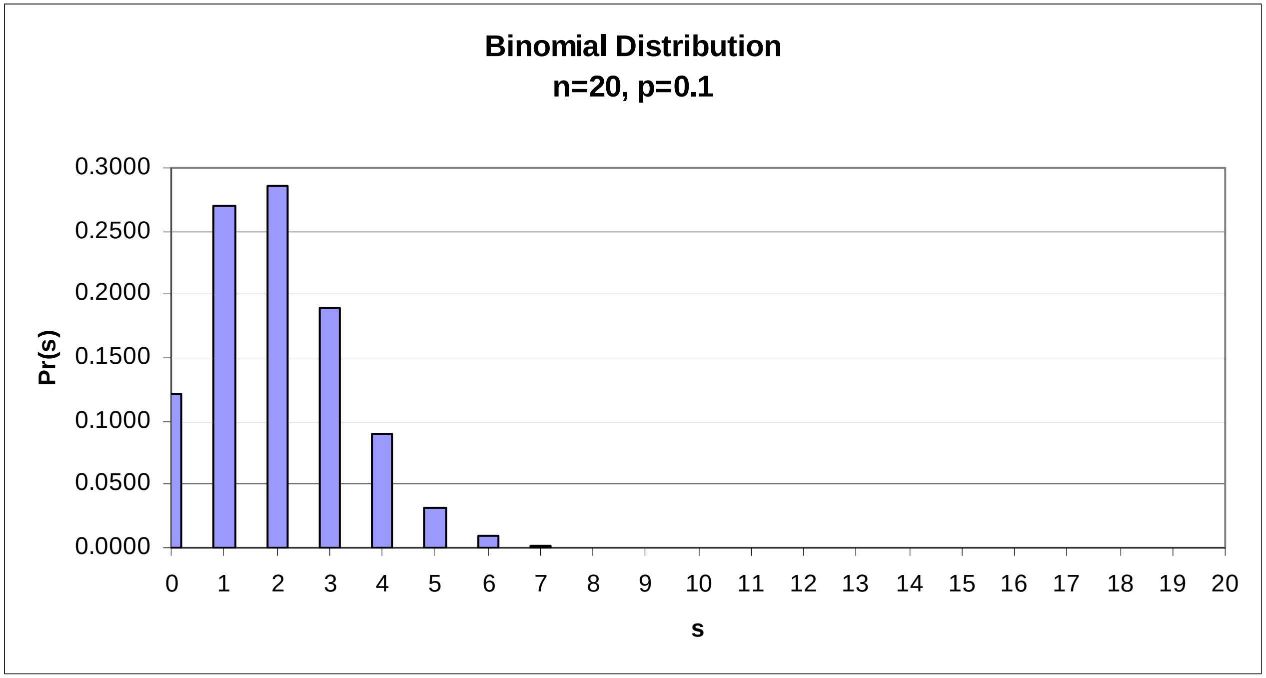
Excel Example

n = 20, *p = 0.1*

What is the probability of exactly 0 successes, 1 success, *etc*.

s	Pr(s)
0	0.1216
1	0.2702
2	0.2852
3	0.1901
4	0.0898
5	0.0319
6	0.0089
7	0.0020
8	0.0004
9	0.0001
10	0.0000
11	0.0000
12	0.0000
. . .	. . .
20	0.0000

BINOMDIST(**number_s**,**trials**,**probability_s**,**cumulative**)



Attribute Sampling Plans

Single Sample Plans

Attribute Sampling Plans

- Single sample plans – Take one sample selected at random and make an accept/reject decision based on the sample
- Double sample plans – Take one sample and make a decision to accept, reject, or take a second sample. If there is second sample, use both to make an accept/reject decision.
- Multiple sample plans – Similar to double sampling, but more than two samples are involved.

The AQL concept

- The AQL is the poorest level of quality (percent nonconforming) that the process can tolerate.

- The input to this process (where I inspect) is defined as:
  - The supplier produces product in lots
  - The supplier uses essentially the same production process for each lot
  - The supplier's production process should run as well as possible, *i.e.,* the process average nonconforming should be as low as possible

- This "poorest level" is the *acceptable quality level* or AQL.

The intentions of the AQL

- The AQL provides a criterion against which to judge lots.

- It does not . . .
  - Provide a process or product specification
  - Allow the supplier to knowingly submit nonconforming product
  - Provide a license to stop continuous improvement activities

The relationship between process control and acceptance sampling

Producer

Production Process

Control Method

SPC: p-chart

Standard given: $p_0 = 0.02$

Central Line: $p_0 = 0.02$

Control Limits:

$$p_0 \pm 3\sqrt{\frac{p_0(1-p_0)}{n}}$$

Consumer

Acceptance Process

Control Method

Attribute Sampling

AQL = 4.0%

Use Z1.4

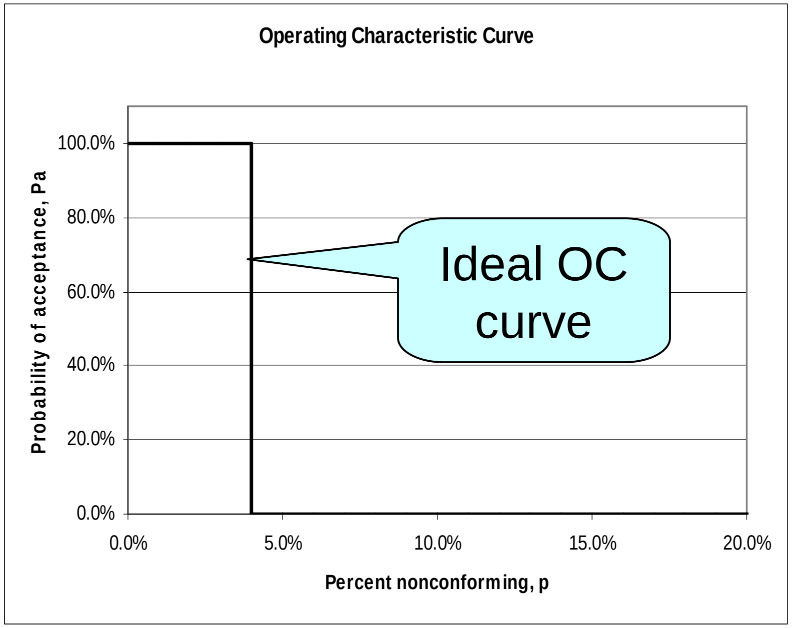
Single Sample

Level II

What Does AQL Mean?

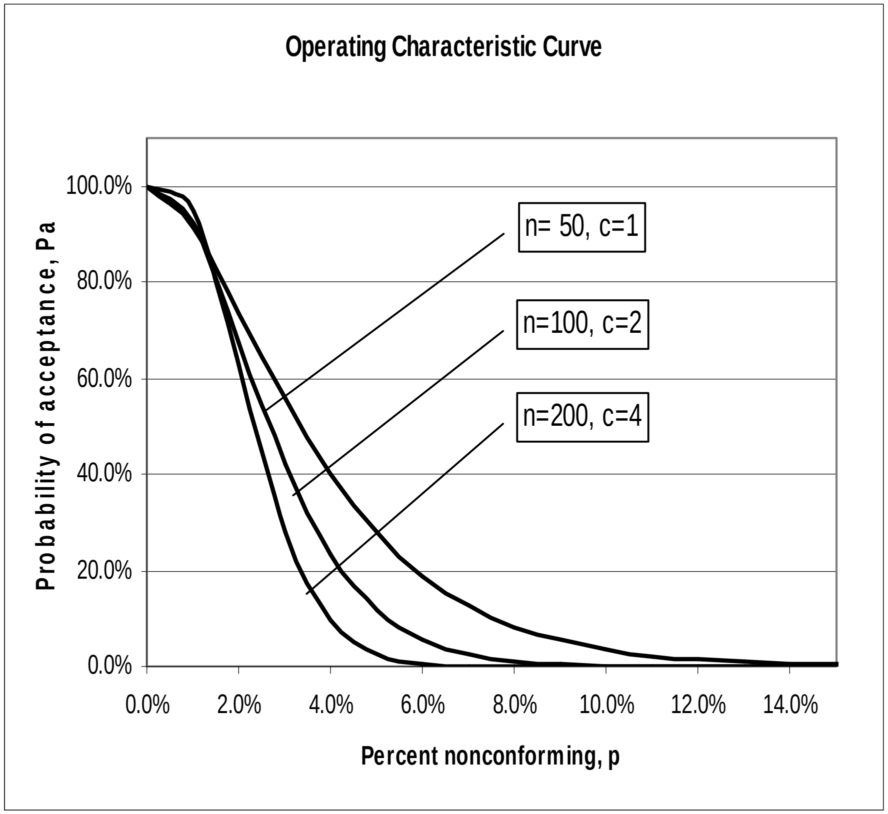
- If the supplier's process average nonconforming is below the AQL, the consumer will accept all the shipped lots.
- If the supplier's process average nonconforming is above the AQL, the consumer will reject all the shipped lots.

Illustrates an AQL of 4.0%

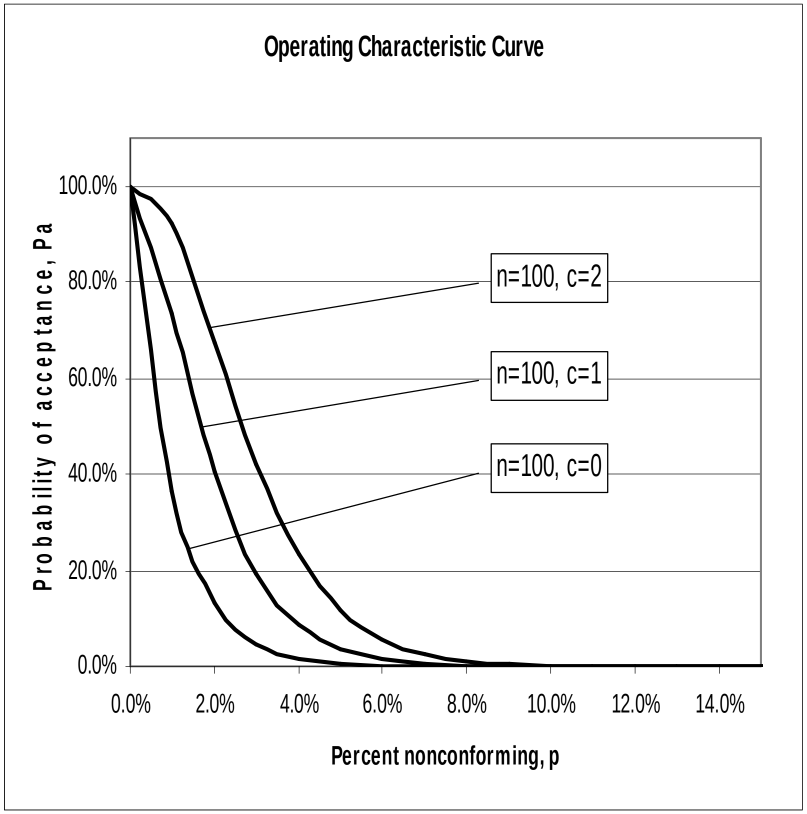


Sampling Doesn't Realize The Ideal OC Curve

Increasing *n* (with *c* proportional) approaches the ideal OC curve.



Increasing *c* (with *n* constant) approaches the ideal OC curve.



Consider Four Possible Outcomes

Producer's Risk – The probability of rejecting a "good" lot.

Consumer's Risk – The probability of accepting a "bad" lot.

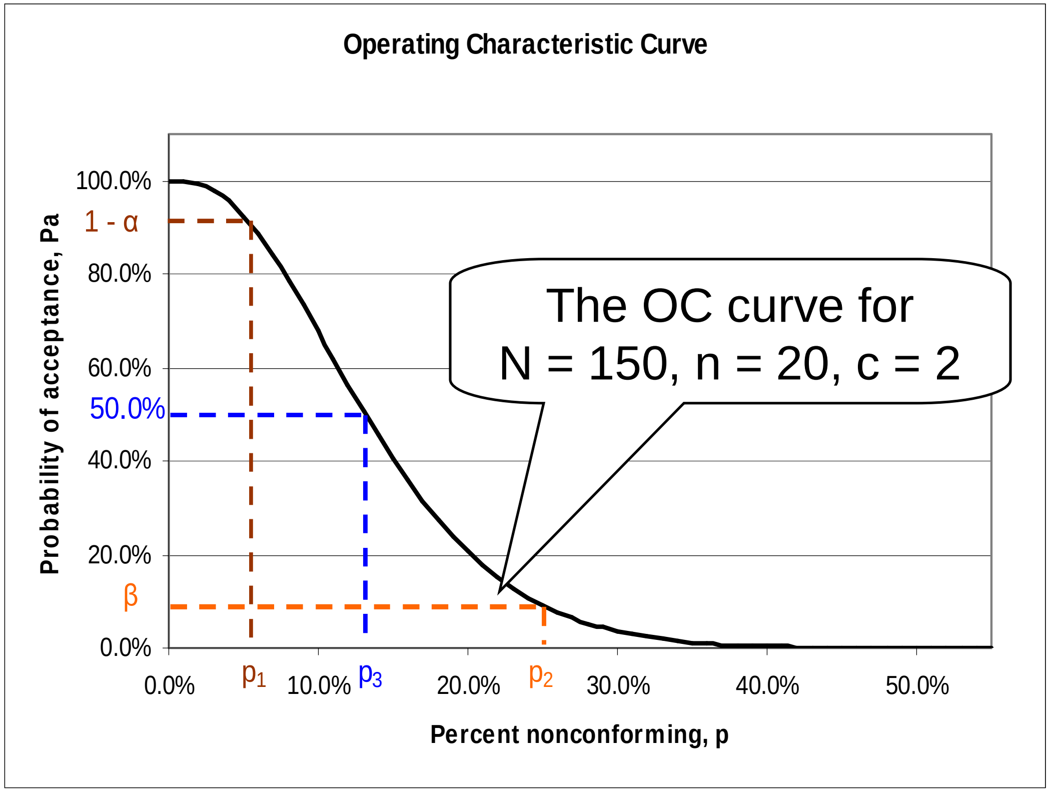
		Consumer's Decision	
		Accept	Reject
Producer's Activity	Lot conforms	OK	Producer's Risk
	Lot doesn't conform	Consumer's Risk	OK

Specific Points on the OC Curve

The Producer's Risk has a value of α.
The point (p_1, 1-α) shows the probability of accepting a lot with quality p_1.

The Consumer's Risk has a value of β.
The point (p_2, β) shows the probability of accepting a lot with quality p_2.

The point (p_3, 0.5) shows the probability of acceptance is 0.5.



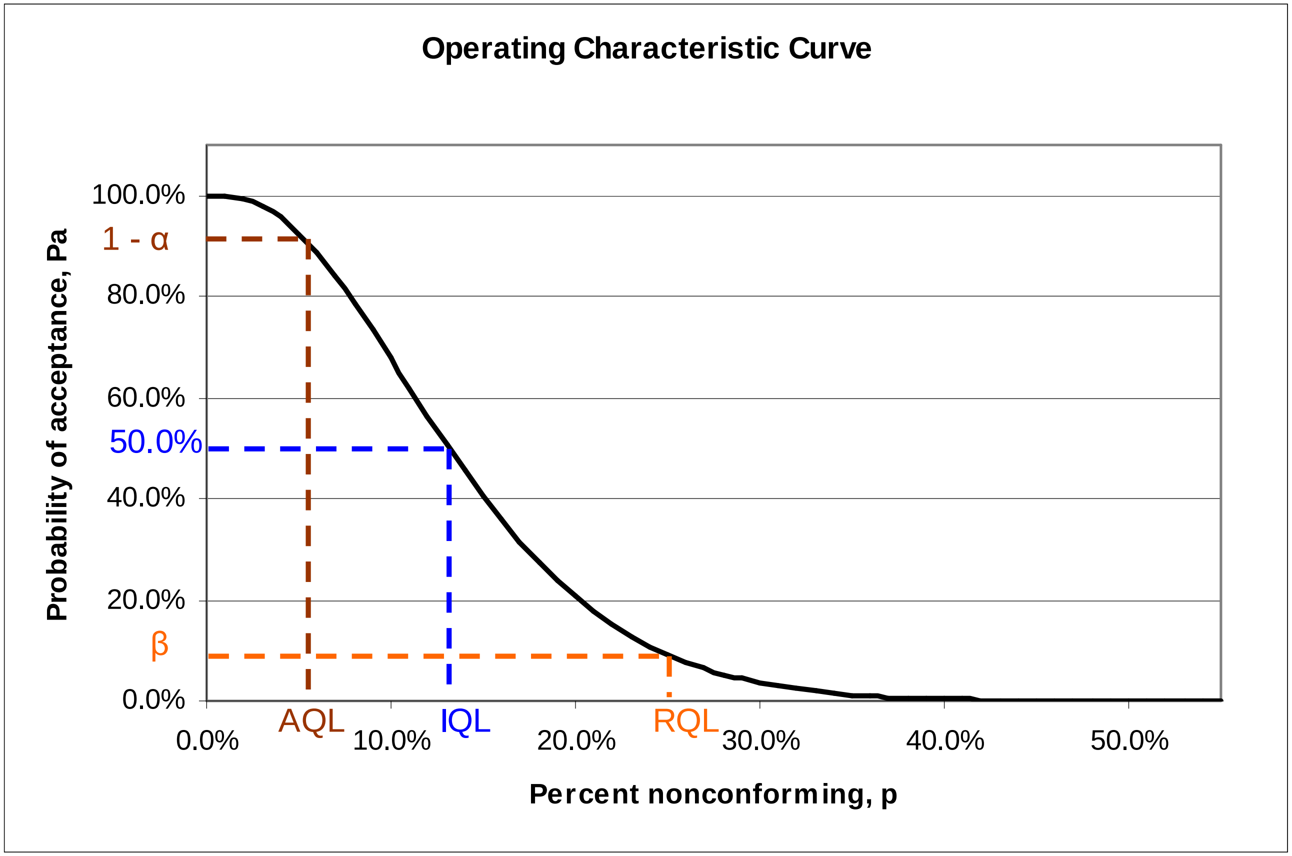
Some Conventions

- Some conventions for these points include:
  - $\alpha = 5\%$ and $\beta = 5\%$
  - The point $(p1, 1-\alpha) = (AQL, 95\%)$
  - The point $(p2, \beta) = (RQL, 5\%)$

- We also see $\alpha = 5\%$ and $\beta = 10\%$
  - The point $(p1, 1-\alpha) = (AQL, 95\%)$
  - The point $(p2, \beta) = (RQL, 10\%)$

- Z1.4 <u>doesn't</u> adopt these conventions

The Previous OC Curve With The Points Named



Characterizing Attribute Sampling Plans

- We typically use four graphs to tell us about a sampling plan.
  - The Operating Characteristic (OC) curve
    - The probability of acceptance for a given quality level.
  - The Average Sample Number (ASN) curve
    - The expected number of items we will sample (most applicable to double, multiple, and sequential samples)
  - The Average Outgoing Quality (AOQ) curve
    - The expected fraction nonconforming after rectifying inspection for a given quality level.
  - The Average Total Inspected (ATI) curve
    - The expected number of units inspected after rectifying inspection for a given quality level.

Rectifying Inspection

- For each lot submitted, we make an accept/reject decision.
  - The accepted lots go to stock
- What do we do with the rejected lots?
  - One solution is to subject them to 100% inspection and replace any nonconforming units with conforming ones.
  - For example, a producer with poor process capability may use this approach.
- Two questions come to mind
  - How many are inspected on average?
  - What happens to outgoing quality after inspection?

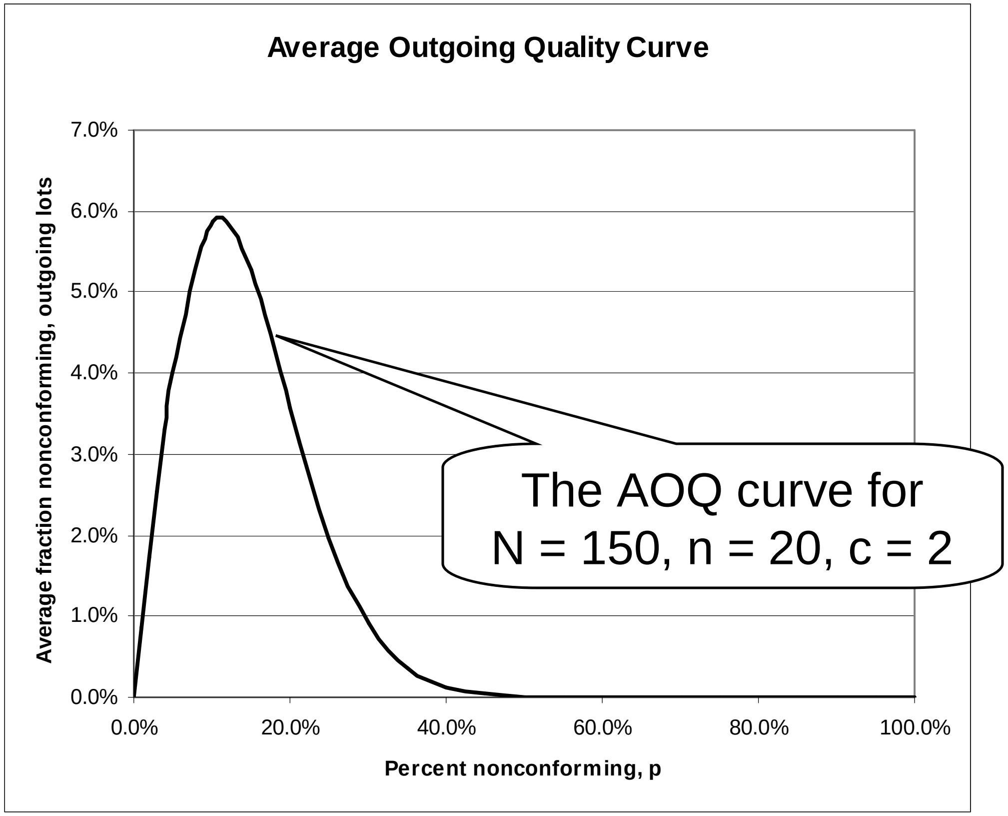
Average Outgoing Quality (AOQ)

Screen the sample
Screen the rejected lots

Screening means to replace all nonconforming units with conforming units.

$$AOQ = \frac{P_a\, p(N-n)}{N}$$

The Average Outgoing Quality Limit (AOQL) is the maximum value of the AOQ



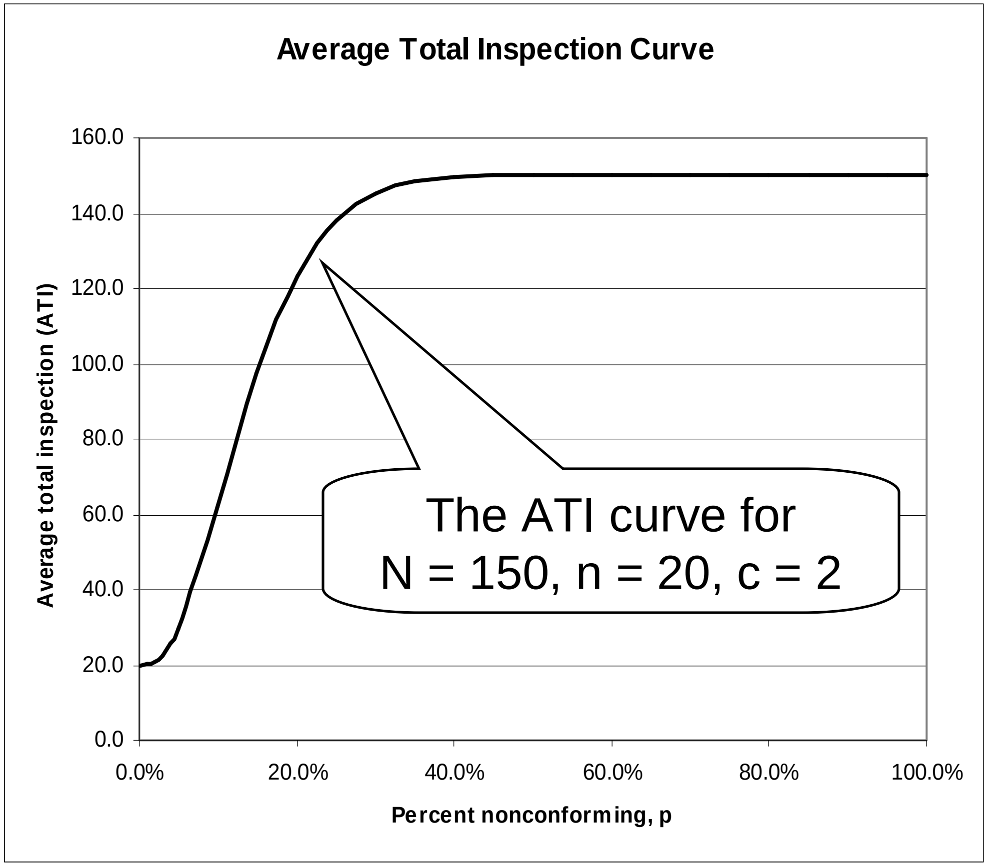
Average Total Inspected (ATI)

If the lot is fully conforming, *p=0.0* (*P_a=1.0*), then we inspect only the sample

If the lot is totally nonconforming, *p=1.0* (*P_a=0.0*), then we inspect the whole lot

$$ATI = n + (1 - P_a)(N - n)$$

For any given lot, we inspect either the sample or the whole lot. On average, we inspect only a portion of the submitted lots

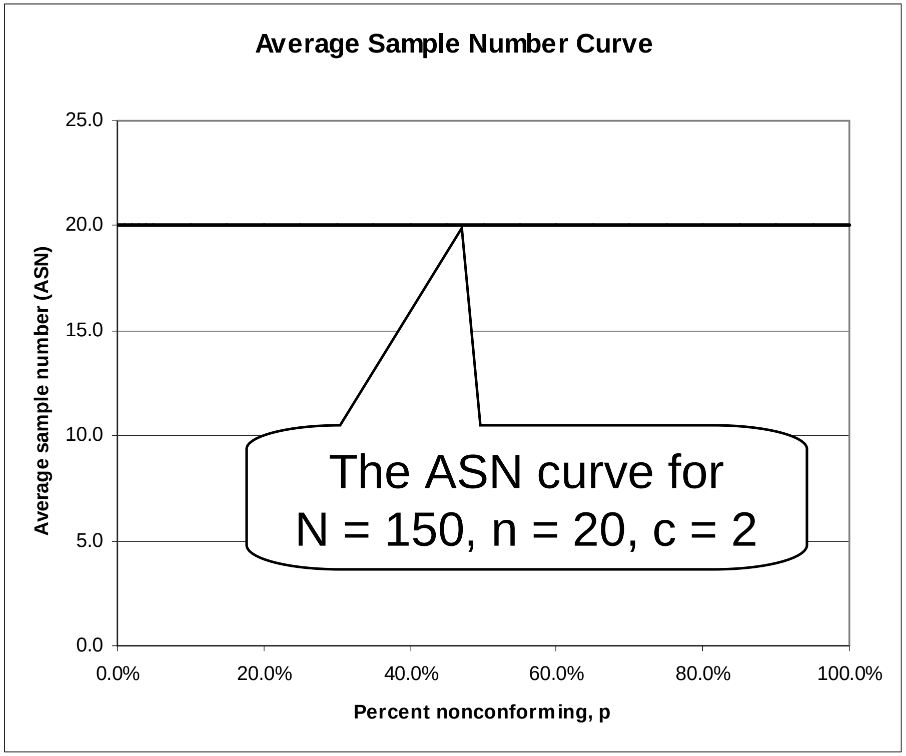


Average Sample Number (ASN)

For single samples, we always inspect the sample.

For double samples, we always inspect the first sample, but sometimes we can make a decision without taking the second sample.

Similarly for multiple samples, we don't always need to take the subsequent samples.



Attribute Sampling Plans

Z1.4 Double Sample Plans

Z1.4 Multiple Sampling Plans

Z1.4 Double Sampling

- Double sampling can reduce the sample size, and thereby reduce cost. (Each double sample is about 62.5% of the single sample.)
- Consider our case: N = 150, AQL = 4.0%
- Table I gives Code letter F
- Table III-A gives the following plan

  $n_1 = 13, c_1 = 0, r_1 = 3$

  $n_2 = 13, c_2 = 3, r_2 = 4$
- On the first sample, we have three possible outcomes: accept, reject, or take the second sample
- On the second sample, we have only two choices, accept or reject.

Switching rules

- The same system of switching rules apply for double and multiple sampling.
- Running a multiple sampling plan system with switching rules can get very confusing.
- The administrative cost goes up along with the potential for error.

Z1.4 Recommendations

- Our recommendation for Z1.4
  - Implement double sampling instead of single sampling.
  - Use the switching rules to get to reduced inspection, again lowering sample sizes.
- Later, we will look at the c=0 plans

Double Sampling Plans

- OC Curve

$$P_a = P(x_1 \le c_1) + \sum_{i=c_1+1}^{r_1-1} P(x_1 = i) P(x_2 \le c_2 - i)$$

- ASN Curve

$$ASN = n_i + n_2 (1 - P_1)$$

- AOQ Curve

$$AOQ = \frac{p \times \left(P_a^1 \times (N - n_1) + P_a^2 \times (N - n_1 - n_2)\right)}{N}$$

- ATI Curve

$$ATI = P_a^1 \times n_1 + P_a^2 \times (n_1 + n_2) + (1 - P_a) \times N$$

P_1 is the probability of making a decision (accept or reject) on the first sample

P_a^i is the probability of acceptance on the i^th^ sample

Attribute Sampling Plans

The c=0 Plans

We look at Squeglia's c=0 plans

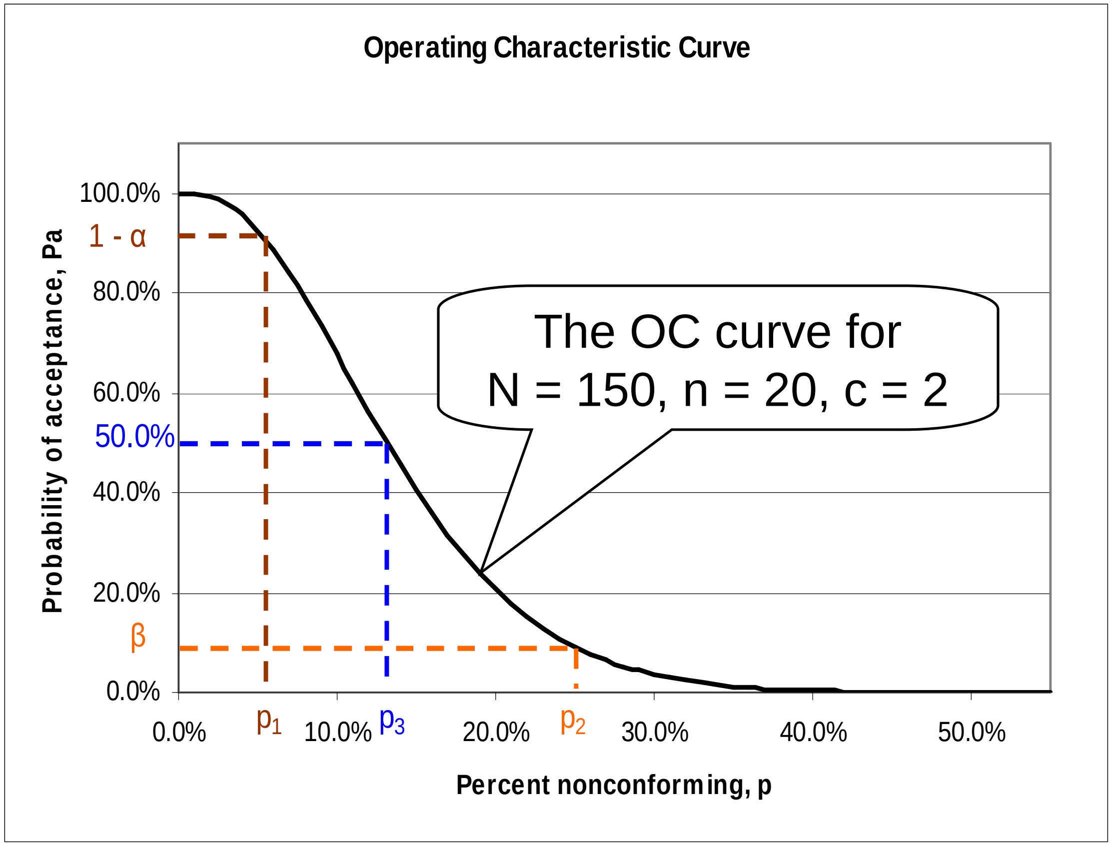
- They are described in *Zero Acceptance Number Sampling Plans*, 5th edition, by Nicholas Squeglia
- They are often called "the c=0 plans"
- The Z1.4 plans tend to look at the AQL
- The c=0 plans look at the LTPD
  - They have (about) the same (LTPD, β) point as the corresponding Z1.4 single normal plan
  - They set $\beta = 0.1$

Recall our earlier discussion of specific points on the OC Curve

The Producer's Risk has a value of α.
The point (p_1, 1-α) shows the probability of accepting a lot with quality p_1.

The Consumer's Risk has a value of β.
The point (p_2, β) shows the probability of accepting a lot with quality p_2.

The point (p_3, 0.5) shows the probability of acceptance is 0.5.



The Difference Between The Plans

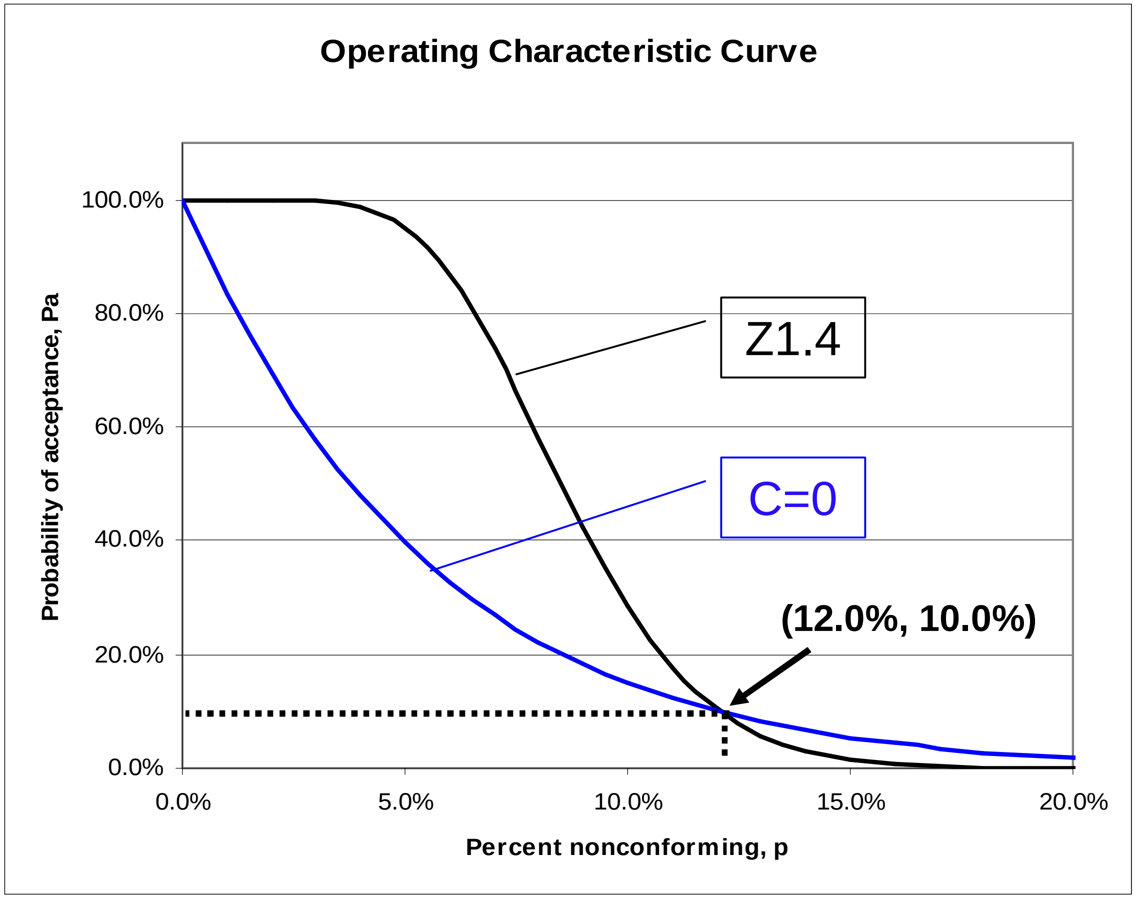
- The c=0 plans are indexed by AQLs to help make them comparable with the Z1.4 plans
- The calculations in the c=0 plan book use the hypergeometric distribution while Z1.4 uses the binomial (and Poisson).
- The c=0 plans try to match the Z1.4 plans at the RQL (or LTPD) point.

Comparison of plans

- An example

  <u>Z1.4:</u>
  N=1300,
  AQL=4.0%,
  n=125,
  c=10

  <u>c=0:</u>
  N=1300
  AQL=4.0%
  n=18
  c=0



Some Things To Observe

- Between 0% nonconforming and the LTPD, the c=0 plan will reject more lots.

- Consider the preceding plan at p = 2.0%
  - P_a for the Z1.4 plan is (nearly) 100%
  - P_a for the c=0 plan is 69.5%

- Hold everything else the same and change from Z1.4 to the corresponding c=0 plan
  - Your inspection costs drop from 125 to 18 pieces
  - Your percentage of <u>rejected</u> lots goes from nearly 0% to about 30%.

c=0 Switching rules

- The c=0 plans don't require switching, but offer it as an option.
  - For tightened go the next lower index (AQL) value
  - For reduced go to the next higher index (AQL) value

- Switching rules

  N $\rightarrow$ T: 2 of 5 rejected

  T $\rightarrow$ N: 5 of 5 accepted

  N $\rightarrow$ R: 10 of 10 accepted

  R $\rightarrow$ N: 1 rejected

Summary and Conclusions

Four Important Curves

- Operating Characteristic (OC)
  - The probability of acceptance as a function of the process nonconformance rate
- Average Sample Number (ASN)
  - The average number of items in the sample(s) as a a function of the process nonconformance rate
  - For single sample plans, it is a constant
- Average Outgoing Quality (AOQ)
  - For rectifying inspection, the quality of the outgoing material
  - The worst case is the Average Outgoing Quality Limit (AOQL)
- Average Total Inspected (ATI)
  - For rectifying inspection, the total number of items inspected a function of the process nonconformance rate

ANSI/ASQ Z1.4

- Offers a huge variety of sampling plans
  - The standard has single, double, and multiple sampling plans
  - The standard includes dynamic adjustments based on the process history (switching rules)
  - The standard offers seven levels for discrimination

- Uses the binomial (or Poisson) distribution

c=0 plans (Squeglia)

- Addresses a common criticism of Z1.4
  - One can accept a lot with nonconforming material in the sample.
- All plans have c=0
  - All OC curves are the special case when c=0
  - The sample sizes tend to be (much) smaller than the corresponding Z1.4 plans
  - Based on the hypergeometric distribution and matched to the Z1.4 plan at the RQL point
  - Indexed by the Z1.4 AQL values for compatibility

Conclusions

- Attribute Sampling is a powerful tool
- There are two common (and many more) sampling plans in use.
  - ANSI/ASQ Z1.4
  - c=0
- Both sets are described by operating characteristic curves
- Deciding factors include the level of protection and the cost

Questions

Attribute Sampling Log
Single Sampling Plan

The product is a wing nut inspected for missing internal threads.

Lot #	Lot Size	Level	Code Letter	State	AQL	Sampling Plan	# Nonconforming Items	Decision	Action
1	5,000	II	L	Normal	1.5	200: 7, 8	10	Reject	
2	900	II	J	Normal	1.5	80: 3, 4	1	Accept	
3	3,000	II	K	Normal	1.5	125: 5, 6	8	Reject	2 of 5 lots rejected Switch to Tightened
4	1,000	II	J	Tightened	1.5	80: 2, 3	1	Accept	
5	1,000	II	J	Tightened	1.5	80: 2, 3	1	Accept	
6	900	II	J	Tightened	1.5	80: 2, 3	0	Accept	
7	2,000	II	K	Tightened	1.5	125: 3, 4	0	Accept	
8	2,500	II	K	Tightened	1.5	125: 3, 4	0	Accept	5 consecutive lots accepted Switch to Normal
9	3,000	II	K	Normal	1.5	125: 5, 6	1	Accept	
10	5,000	II	L	Normal	1.5	200: 7, 8	0	Accept	

Table 1 c=0 sampling plans.

	Index values (associated AQLs)															
	.010	**.015**	**.025**	**.040**	**.065**	**.10**	**.15**	**.25**	**.40**	**.65**	**1.0**	**1.5**	**2.5**	**4.0**	**6.5**	**10.0**
Lot size	**Sample size**															
2–8	*	*	*	*	*	*	*	*	*	*	*	*	5	3	3	3
9–15	*	*	*	*	*	*	*	*	*	*	13	8	5	3	3	3
16–25	*	*	*	*	*	*	*	*	*	20	13	8	5	3	3	3
26–50	*	*	*	*	*	*	*	*	32	20	13	8	7	7	5	3
51–90	*	*	*	*	*	*	80	50	32	20	13	13	11	8	5	4
91–150	*	*	*	*	*	125	80	50	32	20	19	19	11	9	6	5
151–280	*	*	*	*	200	125	80	50	32	29	29	19	13	10	7	6
281–500	*	*	*	315	200	125	80	50	48	47	29	21	16	11	9	7
501–1200	*	800	500	315	200	125	80	75	73	47	34	27	19	15	11	8
1201–3200	1250	800	500	315	200	125	120	116	73	53	42	35	23	18	13	9
3201–10,000	1250	800	500	315	200	192	189	116	86	68	50	38	29	22	15	9
10,001–35,000	1250	800	500	315	300	294	189	135	108	77	60	46	35	29	15	9
35,001–150,000	1250	800	500	490	476	294	218	170	123	96	74	56	40	29	15	9
150,001–500,000	1250	800	750	715	476	345	270	200	156	119	90	64	40	29	15	9
500,001 and over	1250	1200	1112	715	556	435	303	244	189	143	102	64	40	29	15	9

*Indicates entire lot must be inspected.
Note: The acceptance number in all cases is zero.

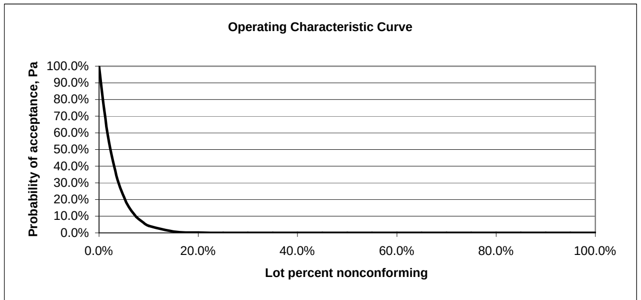
Zero Acceptance Number Sampling Plans, Fifth Edition by Nicholas L. Squeglia
American Society for Quality

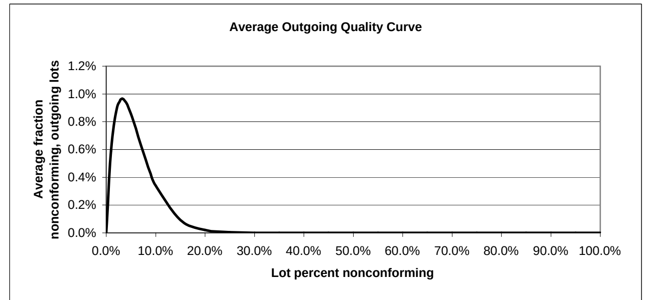
N =	150
n =	30
c =	0

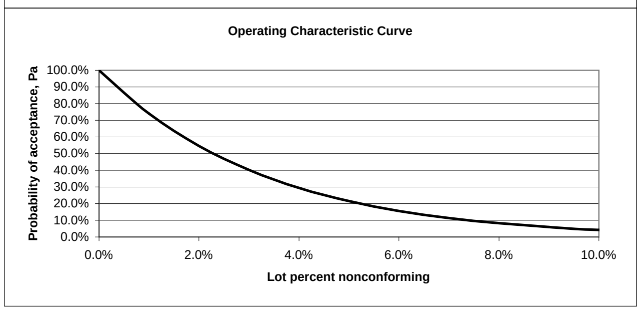
Single Sample Plans

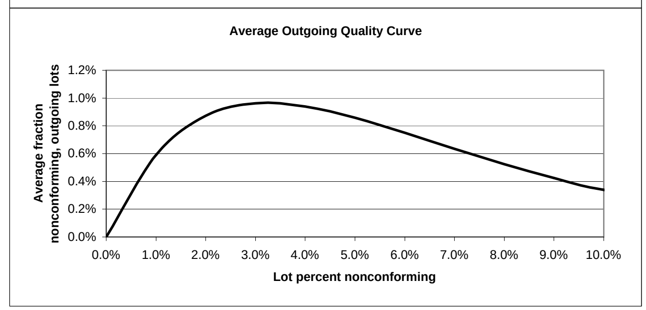
Prepared by Ombu Enterprises, LLC
www.OmbuEnterprises.com
603-209-0600

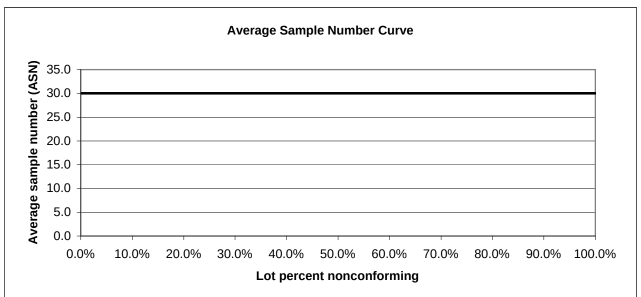
p	Pa	ASN	AOQ	ATI
0.0%	100.0%	30.0	0.0%	30.0
1.0%	74.0%	30.0	0.6%	61.2
2.0%	54.5%	30.0	0.9%	84.5
3.0%	40.1%	30.0	1.0%	101.9
4.0%	29.4%	30.0	0.9%	114.7
5.0%	21.5%	30.0	0.9%	124.2
6.0%	15.6%	30.0	0.8%	131.2
7.0%	11.3%	30.0	0.6%	136.4
8.0%	8.2%	30.0	0.5%	140.2
9.0%	5.9%	30.0	0.4%	142.9
10.0%	4.2%	30.0	0.3%	144.9
15.0%	0.8%	30.0	0.1%	149.1
20.0%	0.1%	30.0	0.0%	149.9
25.0%	0.0%	30.0	0.0%	150.0
30.0%	0.0%	30.0	0.0%	150.0
35.0%	0.0%	30.0	0.0%	150.0
40.0%	0.0%	30.0	0.0%	150.0
45.0%	0.0%	30.0	0.0%	150.0
50.0%	0.0%	30.0	0.0%	150.0
55.0%	0.0%	30.0	0.0%	150.0
60.0%	0.0%	30.0	0.0%	150.0
65.0%	0.0%	30.0	0.0%	150.0
70.0%	0.0%	30.0	0.0%	150.0
75.0%	0.0%	30.0	0.0%	150.0
80.0%	0.0%	30.0	0.0%	150.0
85.0%	0.0%	30.0	0.0%	150.0
90.0%	0.0%	30.0	0.0%	150.0
95.0%	0.0%	30.0	0.0%	150.0
100.0%	0.0%	30.0	0.0%	150.0

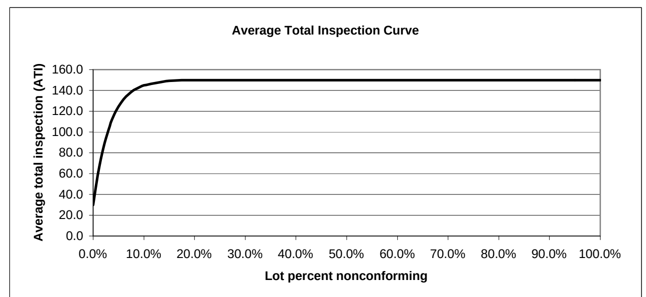


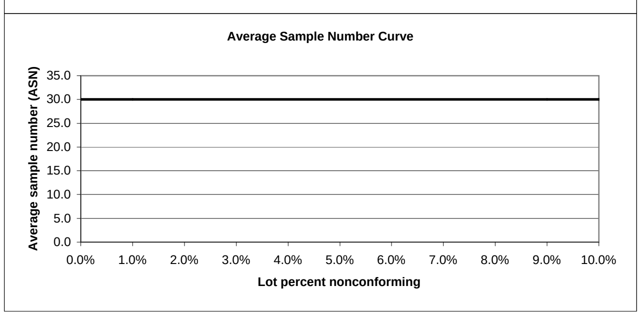


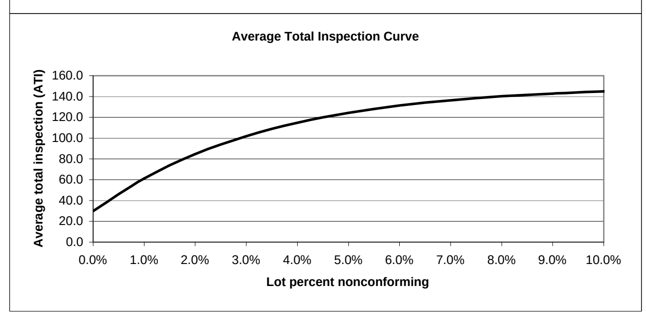










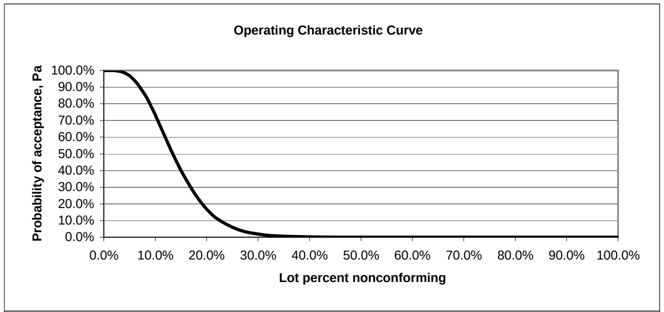


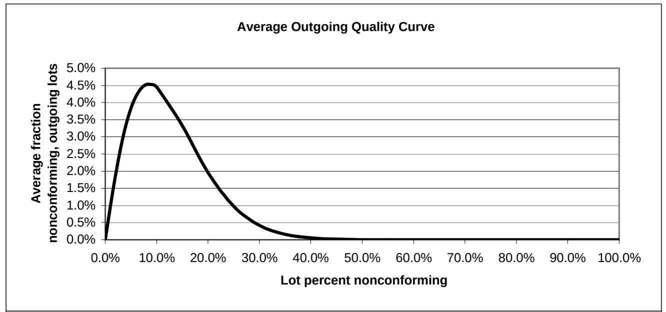
N =	1,500	
n =	18	18
c =	1	4
r =	4	5

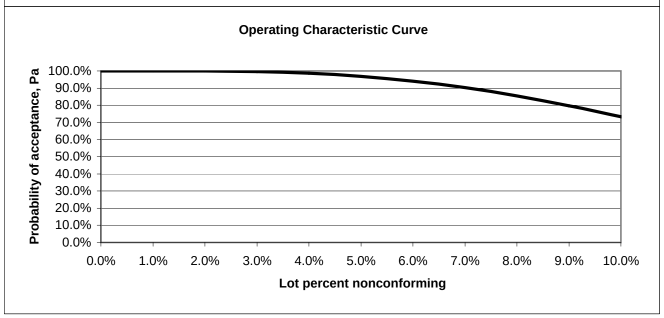
Double Sample Plans

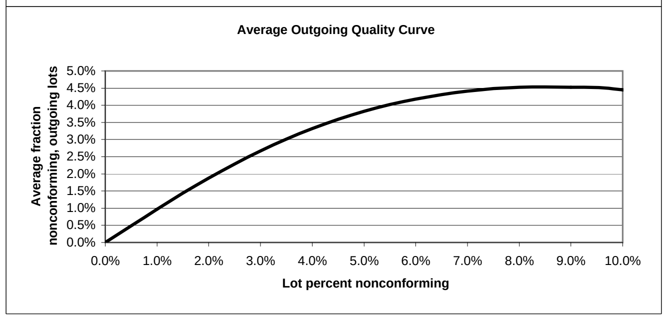
Prepared by Ombu Enterprises, LLC
www.OmbuEnterprises.com
603-209-0600

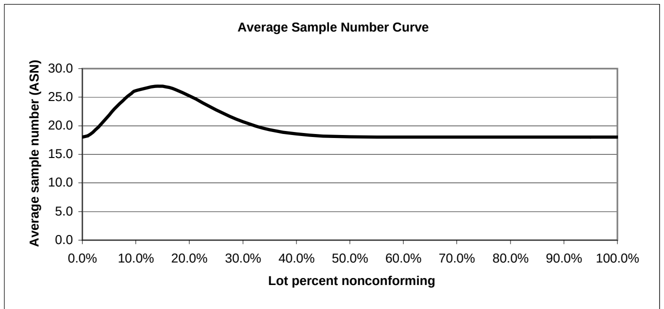
p	Pa	ASN	AOQ	ATI
0.0%	100.0%	18.0	0.0%	18.0
1.0%	100.0%	18.2	1.0%	18.3
2.0%	99.9%	18.9	1.9%	20.2
3.0%	99.6%	19.8	2.7%	26.4
4.0%	98.6%	20.8	3.3%	41.1
5.0%	96.9%	21.9	3.8%	68.0
6.0%	94.1%	22.9	4.2%	109.7
7.0%	90.3%	23.9	4.4%	167.0
8.0%	85.4%	24.8	4.5%	239.0
9.0%	79.7%	25.5	4.5%	323.5
10.0%	73.4%	26.1	4.4%	417.4
15.0%	40.1%	26.9	3.3%	909.5
20.0%	16.9%	25.2	2.0%	1251.4
25.0%	5.9%	22.8	1.0%	1412.7
30.0%	1.8%	20.7	0.4%	1472.8
35.0%	0.5%	19.3	0.2%	1492.2
40.0%	0.1%	18.6	0.1%	1497.9
45.0%	0.0%	18.2	0.0%	1499.5
50.0%	0.0%	18.1	0.0%	1499.9
55.0%	0.0%	18.0	0.0%	1500.0
60.0%	0.0%	18.0	0.0%	1500.0
65.0%	0.0%	18.0	0.0%	1500.0
70.0%	0.0%	18.0	0.0%	1500.0
75.0%	0.0%	18.0	0.0%	1500.0
80.0%	0.0%	18.0	0.0%	1500.0
85.0%	0.0%	18.0	0.0%	1500.0
90.0%	0.0%	18.0	0.0%	1500.0
95.0%	0.0%	18.0	0.0%	1500.0
100.0%	0.0%	18.0	0.0%	1500.0

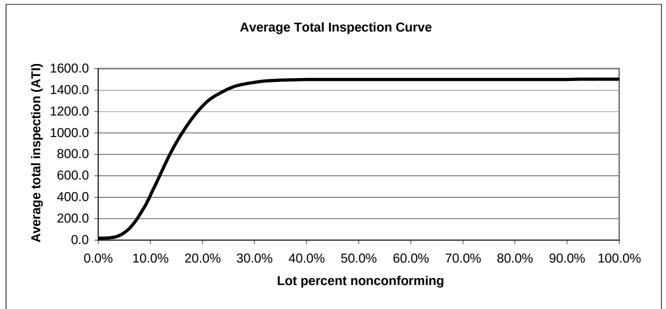


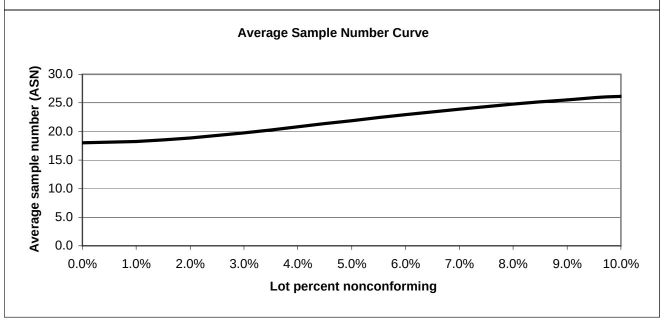


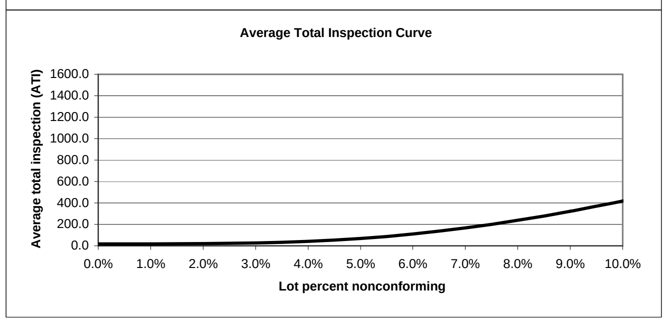












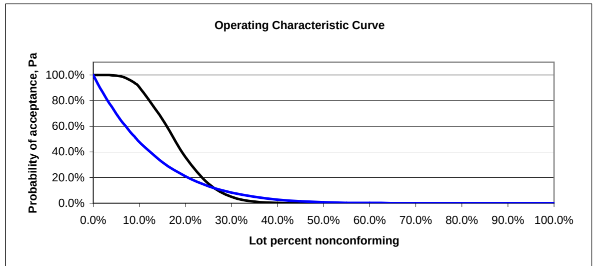
Z1.4	
AQL	6.5%
N =	200
n =	32
c =	5

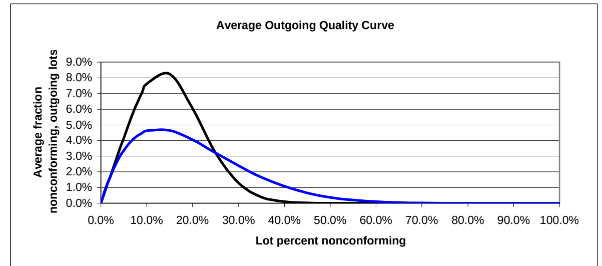
c=0	
AQL	6.5%
N =	200
n =	7
c =	0

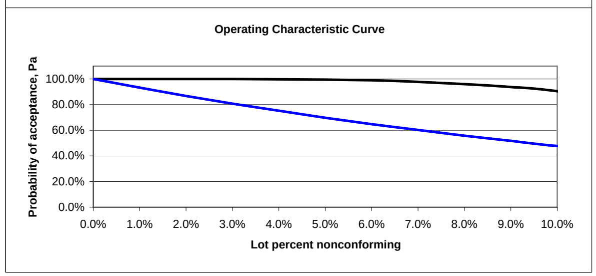
Z1.4 v c=0

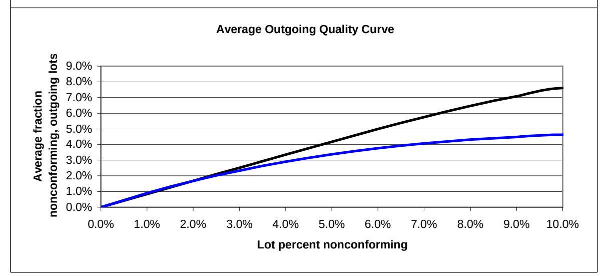
Prepared by Ombu Enterprises, LLC
www.OmbuEnterprises.com
603-209-0600

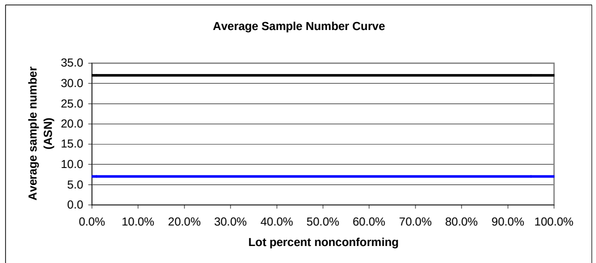
	Z1.4				c=0			
p	Pa	AOQ	ASN	ATI	Pa	AOQ	ASN	ATI
0.0%	100.0%	0.0%	32.0	32.0	100.0%	0.0%	7.0	7.0
1.0%	100.0%	0.8%	32.0	32.0	93.2%	0.9%	7.0	20.1
2.0%	100.0%	1.7%	32.0	32.0	86.8%	1.7%	7.0	32.5
3.0%	100.0%	2.5%	32.0	32.1	80.8%	2.3%	7.0	44.1
4.0%	99.8%	3.4%	32.0	32.3	75.1%	2.9%	7.0	55.0
5.0%	99.5%	4.2%	32.0	32.8	69.8%	3.4%	7.0	65.2
6.0%	98.9%	5.0%	32.0	33.8	64.8%	3.8%	7.0	74.8
7.0%	97.8%	5.8%	32.0	35.7	60.2%	4.1%	7.0	83.9
8.0%	96.1%	6.5%	32.0	38.6	55.8%	4.3%	7.0	92.3
9.0%	93.7%	7.1%	32.0	42.6	51.7%	4.5%	7.0	100.3
10.0%	90.6%	7.6%	32.0	47.9	47.8%	4.6%	7.0	107.7
15.0%	65.4%	8.2%	32.0	90.1	32.1%	4.6%	7.0	138.1
20.0%	36.0%	6.1%	32.0	139.5	21.0%	4.0%	7.0	159.5
25.0%	15.3%	3.2%	32.0	174.3	13.3%	3.2%	7.0	174.2
30.0%	5.1%	1.3%	32.0	191.4	8.2%	2.4%	7.0	184.1
35.0%	1.3%	0.4%	32.0	197.7	4.9%	1.7%	7.0	190.5
40.0%	0.3%	0.1%	32.0	199.5	2.8%	1.1%	7.0	194.6
45.0%	0.0%	0.0%	32.0	199.9	1.5%	0.7%	7.0	197.1
50.0%	0.0%	0.0%	32.0	200.0	0.8%	0.4%	7.0	198.5
55.0%	0.0%	0.0%	32.0	200.0	0.4%	0.2%	7.0	199.3
60.0%	0.0%	0.0%	32.0	200.0	0.2%	0.1%	7.0	199.7
65.0%	0.0%	0.0%	32.0	200.0	0.1%	0.0%	7.0	199.9
70.0%	0.0%	0.0%	32.0	200.0	0.0%	0.0%	7.0	200.0
75.0%	0.0%	0.0%	32.0	200.0	0.0%	0.0%	7.0	200.0
80.0%	0.0%	0.0%	32.0	200.0	0.0%	0.0%	7.0	200.0
85.0%	0.0%	0.0%	32.0	200.0	0.0%	0.0%	7.0	200.0
90.0%	0.0%	0.0%	32.0	200.0	0.0%	0.0%	7.0	200.0
95.0%	0.0%	0.0%	32.0	200.0	0.0%	0.0%	7.0	200.0
100.0%	0.0%	0.0%	32.0	200.0	0.0%	0.0%	7.0	200.0

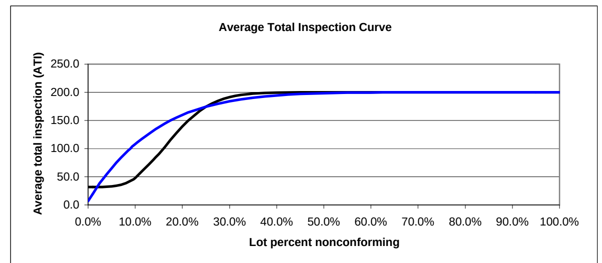


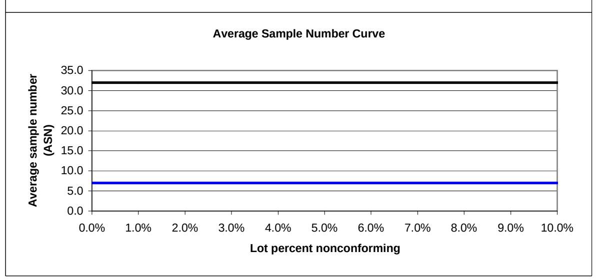












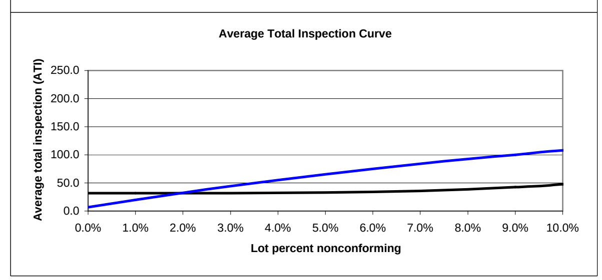


Table III-A—Double sampling plans for normal inspection (Master table)

(See 9.4 and 9.5)

Sample size code letter	Sample	Sample size	Cumulative sample size	Acceptable Quality Levels (normal inspection)																									
				0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
				Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A																	↓	*		↓	*	*	*	*	*	*	*	*	*
B	First	2	2													↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	17 22	25 31
	Second	2	4																	1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27	37 38	56 57
C	First	3	3												↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	17 22	25 31	↑
	Second	3	6																1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27	37 38	56 57	
D	First	5	5											↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	17 22	25 31	↑	
	Second	5	10															1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27	37 38	56 57		
E	First	8	8										↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	17 22	25 31	↑		
	Second	8	16														1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27	37 38	56 57			
F	First	13	13									↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑	↑	↑			
	Second	13	26													1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27						
G	First	20	20								↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑						
	Second	20	40												1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27							
H	First	32	32							↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑							
	Second	32	64											1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27								
J	First	50	50						↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑								
	Second	50	100										1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27									
K	First	80	80					↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑									
	Second	80	160									1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27										
L	First	125	125				↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑										
	Second	125	250								1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27											
M	First	200	200			↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑											
	Second	200	400							1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27												
N	First	315	315		↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑												
	Second	315	630						1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27													
P	First	500	500	↓	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑													
	Second	500	1000					1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27														
Q	First	800	800	*	↑	↓	0 2	0 3	1 4	2 5	3 7	5 9	5 9	11 16	↑														
	Second	800	1600				1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27															
R	First	1250	1250	↑	↑	0 2	0 3	1 4	2 5	3 7	5 9	7 11	11 16	↑															
	Second	1250	2500			1 2	3 4	4 5	6 7	8 9	12 13	18 19	26 27																

↓ = Use first sampling plan below arrow. If sample size equals or exceeds lot or batch size, do 100 percent inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

* = Use corresponding single sampling plan (or alternatively, use double sampling plan below, where available).

Table II-B—Single sampling plans for tightened inspection (Master table)

(See 9.4 and 9.5)

Sample size code letter	Sample size	Acceptable Quality Levels (tightened inspection)																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	0 1	↑	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
S	3150		↑	1 2								↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow. If sample size equals or exceeds lot or batch size, do 100 percent inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

Table II-A—Single sampling plans for normal inspection (Master table)

(See 9.4 and 9.5)

Acceptable Quality Levels (normal inspection)

Sample size code letter	Sample size	0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓
H	50	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2
J	80	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3
K	125	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4
L	200	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6
M	315	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8
N	500	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11
P	800	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15
Q	1250	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22
R	2000	↑	↑	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑

Sample size code letter	Sample size	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	3	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45
C	5	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑
D	8	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑
E	13	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑	↑
F	20	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑
G	32	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑
H	50	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑
J	80	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100 percent inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

TABLE I—Sample size code letters

(See 9.2 and 9.3)

Lot or batch size			Special inspection levels				General inspection levels		
			S-1	S-2	S-3	S-4	I	II	III
2	to	8	A	A	A	A	A	A	B
9	to	15	A	A	A	A	A	B	C
16	to	25	A	A	B	B	B	C	D
26	to	50	A	B	B	C	C	D	E
51	to	90	B	B	C	C	C	E	F
91	to	150	B	B	C	D	D	F	G
151	to	280	B	C	D	E	E	G	H
281	to	500	B	C	D	E	F	H	J
501	to	1200	C	C	E	F	G	J	K
1201	to	3200	C	D	E	G	H	K	L
3201	to	10000	C	D	F	G	J	L	M
10001	to	35000	C	D	F	H	K	M	N
35001	to	150000	D	E	G	J	L	N	P
150001	to	500000	D	E	G	J	M	P	Q
500001	and	over	D	E	H	K	N	Q	R

TABLE I—Sample size code letters

(See 9.2 and 9.3)

Lot or batch size			Special inspection levels				General inspection levels		
			S-1	S-2	S-3	S-4	I	II	III
2	to	8	A	A	A	A	A	A	B
9	to	15	A	A	A	A	A	B	C
16	to	25	A	A	B	B	B	C	D
26	to	50	A	B	B	C	C	D	E
51	to	90	B	B	C	C	C	E	F
91	to	150	B	B	C	D	D	F	G
151	to	280	B	C	D	E	E	G	H
281	to	500	B	C	D	E	F	H	J
501	to	1200	C	C	E	F	G	J	K
1201	to	3200	C	D	E	G	H	K	L
3201	to	10000	C	D	F	G	J	L	M
10001	to	35000	C	D	F	H	K	M	N
35001	to	150000	D	E	G	J	L	N	P
150001	to	500000	D	E	G	J	M	P	Q
500001	and	over	D	E	H	K	N	Q	R

TABLE I—Sample size code letters

(See 9.2 and 9.3)

Lot or batch size			Special inspection levels				General inspection levels		
			S-1	S-2	S-3	S-4	I	II	III
2	to	8	A	A	A	A	A	A	B
9	to	15	A	A	A	A	A	B	C
16	to	25	A	A	B	B	B	C	D
26	to	50	A	B	B	C	C	D	E
51	to	90	B	B	C	C	C	E	F
91	to	150	B	B	C	D	D	F	G
151	to	280	B	C	D	E	E	G	H
281	to	500	B	C	D	E	F	H	J
501	to	1200	C	C	E	F	G	J	K
1201	to	3200	C	D	E	G	H	K	L
3201	to	10000	C	D	F	G	J	L	M
10001	to	35000	C	D	F	H	K	M	N
35001	to	150000	D	E	G	J	L	N	P
150001	to	500000	D	E	G	J	M	P	Q
500001	and	over	D	E	H	K	N	Q	R

TABLE I—Sample size code letters

(See 9.2 and 9.3)

Lot or batch size			Special inspection levels				General inspection levels		
			S-1	S-2	S-3	S-4	I	II	III
2	to	8	A	A	A	A	A	A	B
9	to	15	A	A	A	A	A	B	C
16	to	25	A	A	B	B	B	C	D
26	to	50	A	B	B	C	C	D	E
51	to	90	B	B	C	C	C	E	F
91	to	150	B	B	C	D	D	F	G
151	to	280	B	C	D	E	E	G	H
281	to	500	B	C	D	E	F	H	J
501	to	1200	C	C	E	F	G	J	K
1201	to	3200	C	D	E	G	H	K	L
3201	to	10000	C	D	F	G	J	L	M
10001	to	35000	C	D	F	H	K	M	N
35001	to	150000	D	E	G	J	L	N	P
150001	to	500000	D	E	G	J	M	P	Q
500001	and	over	D	E	H	K	N	Q	R