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Process Capability Indices:

"The Good, The Bad, and The Truly Ugly"

ASQ Granite State Section
Monthly Meeting 1/20/09

Presented by the North Haven Group, LLC

Philip J. Ramsey, Ph.D. – pramsey@northhavengroup.com

Mia L. Stephens, M.S. – mstephens@northhavengroup.com

Marie A. Gaudard, Ph.D. – mgaudard@northhavengroup.com

Talk Outline

- Introduction
- What is Process Capability?
- Process Capability Indices
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- Short Term vs. Long Term Variation
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- Confidence Intervals for Cpk
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Introduction

A process is said to be **stable** or **in statistical control** when special causes don’t exist, namely, when only common causes of variation appear to be present.

However, a stable process can be producing defects at a highly unacceptable rate.

A process can be in statistical control and not be capable of producing the desired quality relative to the specification limits.

Stability says nothing about performance.

Process capability studies are used to assess the performance of the process with respect to specification limits.

Introduction

In order to evaluate the capability of a process, we need to assume that the process is stable and that the measurement system is capable.

We consider four issues:

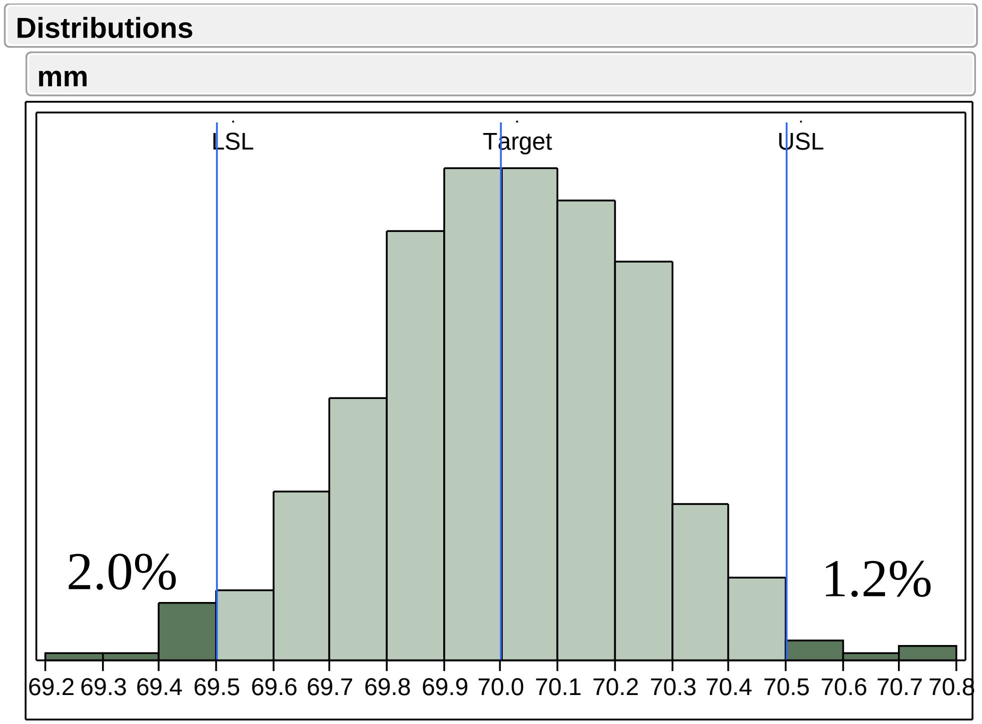
- **Is the process on target?** Is the process centered at its target?
- **Is the process consistent**? How much spread is there in the quality characteristic?
- **How does the process distribution compare to the specification limits**? What percentage of parts will be "out of spec"?
- **What is the shape of the distribution**?

What is Process Capability?

For a piston ring grinding process, the specifications for diameter are 70 ± 0.5 mm.

For 500 measurements, the mean is 70.01 mm and the standard deviation is 0.232 mm.

- Is the process on target?
- Is the process consistent?
- What percentage of values fall outside the spec limits?
- What is the shape of the distribution?



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What is Process Capability?

Process capability, as a generic term, refers to the spread (variability) of a stable process.

Process capability indices are measures that relate that spread to the specification limits.

A **Process Capability Study** determines whether a process is unstable, investigates sources of instability, and determines their causes.

After action is taken to eliminate the sources of instability, the process's capability can be determined.

Process capability studies compare process outputs to customer requirements or expectations.

What is Process Capability?

There are two types of process capability studies:

- **Attribute capability studies** (e.g., go, no-go data) and
- **Variables capability studies** (continuous measurements).

Attributes Process Capability Studies determine process capability in terms of **proportion defective**.

Attribute measures, which are often used to monitor characteristics that are difficult to measure continuously, are seriously limiting in improvement studies for a number of reasons.

Attribute process capability studies require a great deal of data.

Our focus in this talk is on variables capability studies.

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What is Process Capability?

Variables Process Capability Studies determine process capability in terms of the distribution of process output in relation to specification limits.

Some of the advantages of these studies over attributes studies are:

- They address the **stability** of the process in an efficient way,
- They are **sensitive to shifts** in the process,
- They provide information regarding the **centering** and **variability** of the process, and
- They allow for **examination of specification limits** to determine whether the limits were reasonable in the first place.

What is Process Capability?

Variables control charts, such as X-bar and R charts, are used to stabilize a process prior to determining capability.

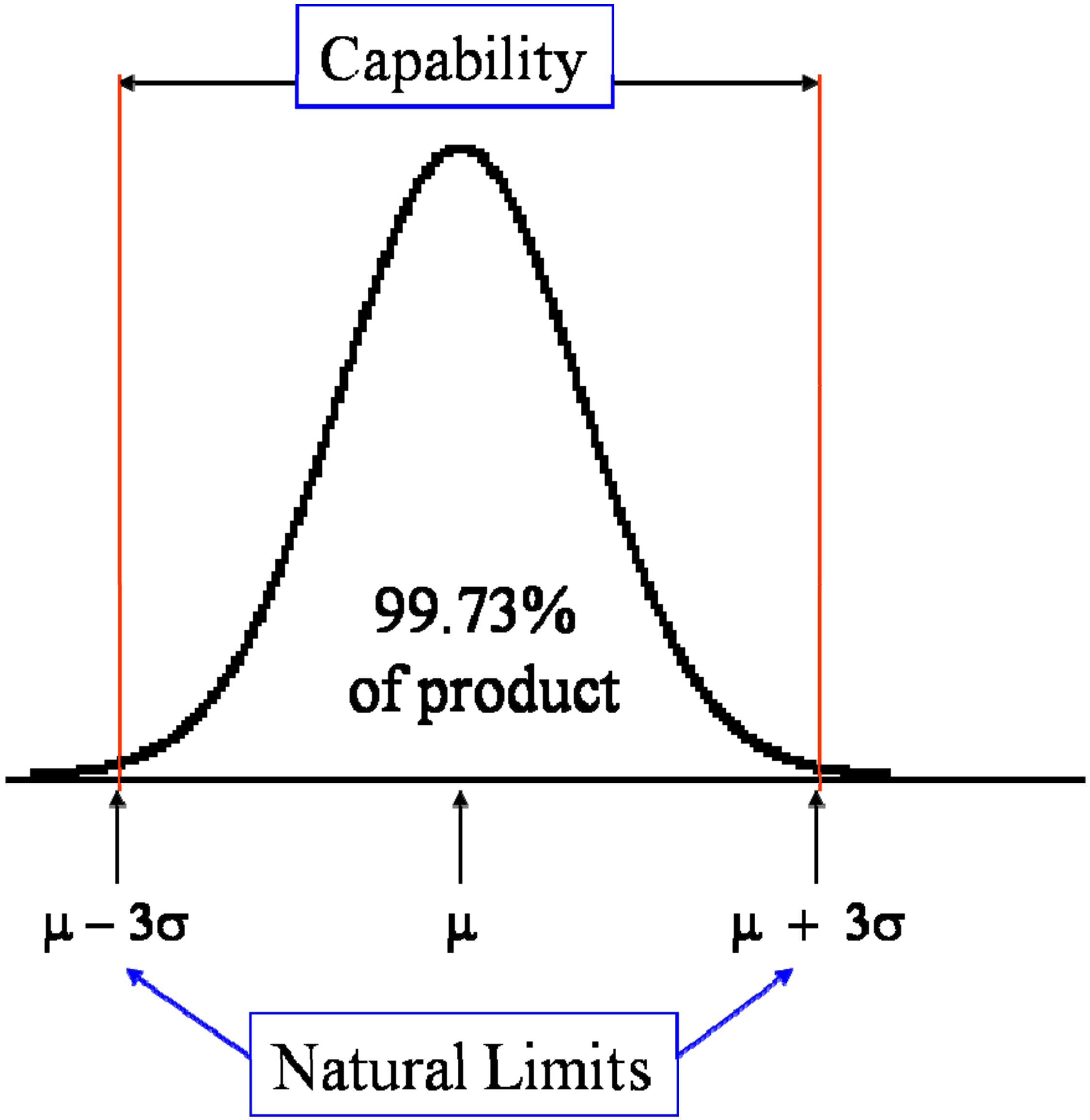
Once sources of special cause variation have been eliminated, we can determine the **natural limits** for our process.

Not to be confused with control limits, natural limits apply to **individual units** of output rather than subgroups.

If the output is stable and normally distributed, then approximately 99.73% of all process output will fall between the natural limits.

In contrast, approximately 99.73% of all subgroup means will fall within the control limits.

What is Process Capability?



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What is Process Capability?

Inherent Process Variation (short term variation), or common cause variation, is used to calculate natural limits.

We usually estimate short term variation, $\boldsymbol{\sigma_{ST}}$, by using the center line of an R or S chart constructed using **rational subgroups,** or, for individual measurements, a Moving Range chart constructed over a short time interval.

Process Capability (sometimes called **short term capability** or **potential capability**) is defined as the 6σ interval for a process based upon this short term estimate of σ (σ_{ST}).

Almost all of the output of a stable process (99.73%) will fall within this 6σ interval.

Process Capability Indices

Process capability indices quantify the ability of a process to meet specifications.

All of the process capability indices we will discuss require us to meet the following **basic assumptions**:

- **Stability** - the process must be stable;
- **Continuous data** must be used;
- The process characteristic under study must be **approximately normal** (this is a fairly strong assumption);
- Observations are **independent** and, in particular, not **auto-correlated** (this is a very strong assumption);
- **Two-sided specification limits are symmetric –** an absolute necessity in order to use process capability indices.

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Process Capability Indices

Four short-term capability indices are commonly used:

- C_P measures the ability to meet two-sided specification limits, and assumes that the process is centered on the target,
- C_{PU} measures the ability to meet a one-sided upper spec,
- C_{PL} measures the ability to meet a one-sided lower spec, and
- C_{PK} attempts to account for off target performance relative to two-sided specification limits.

Although other capability indices are sometimes used, we will limit our discussion to those listed above.

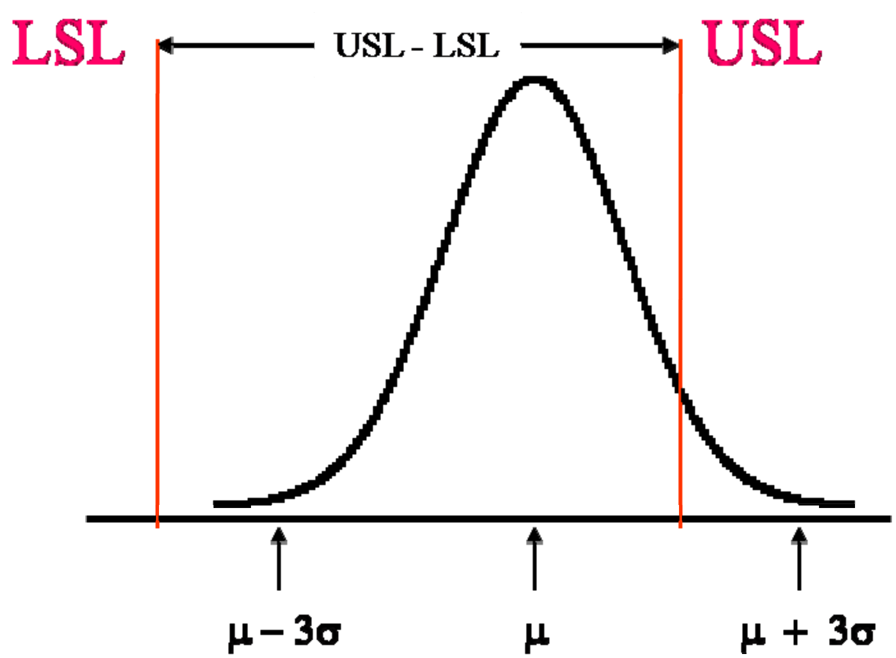
Process Capability Indices

The C_P index is the **ratio of the tolerance interval to the process performance.**

$$C_p = \frac{USL - LSL}{6\sigma_{ST}}$$

← Tolerance Interval

← Process Performance

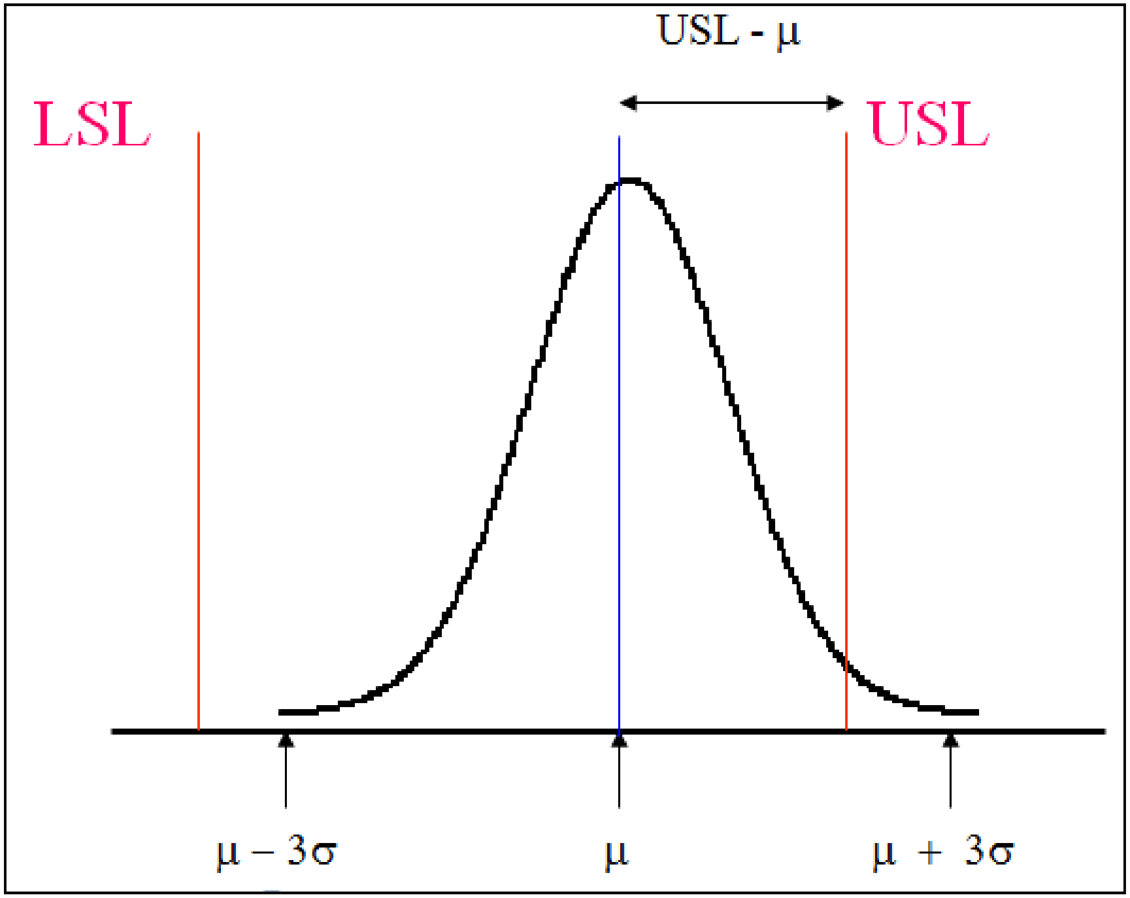


C_P does not account for off-target behavior.

Process Capability Indices

C_{PU} is used to examine the ability to meet the **upper specification limit**, or when only an upper specification limit exists.

$$C_{PU} = \frac{USL - \mu}{3\sigma_{ST}}$$



Process Capability Indices

C_{PL} is used to examine the ability to meet the **lower specification limit**, or when only a lower specification limit exists.

$$C_{PU} = \frac{USL - \mu}{3\sigma_{ST}}; \quad C_{PL} = \frac{\mu - LSL}{3\sigma_{ST}}$$

C_{PK} attempts to **account for off-target performance** for the process. C_{PL} and the C_{PU} are be used to calculate C_{PK}:

$$C_{PK} = \min\{C_{PU}, C_{PL}\}$$

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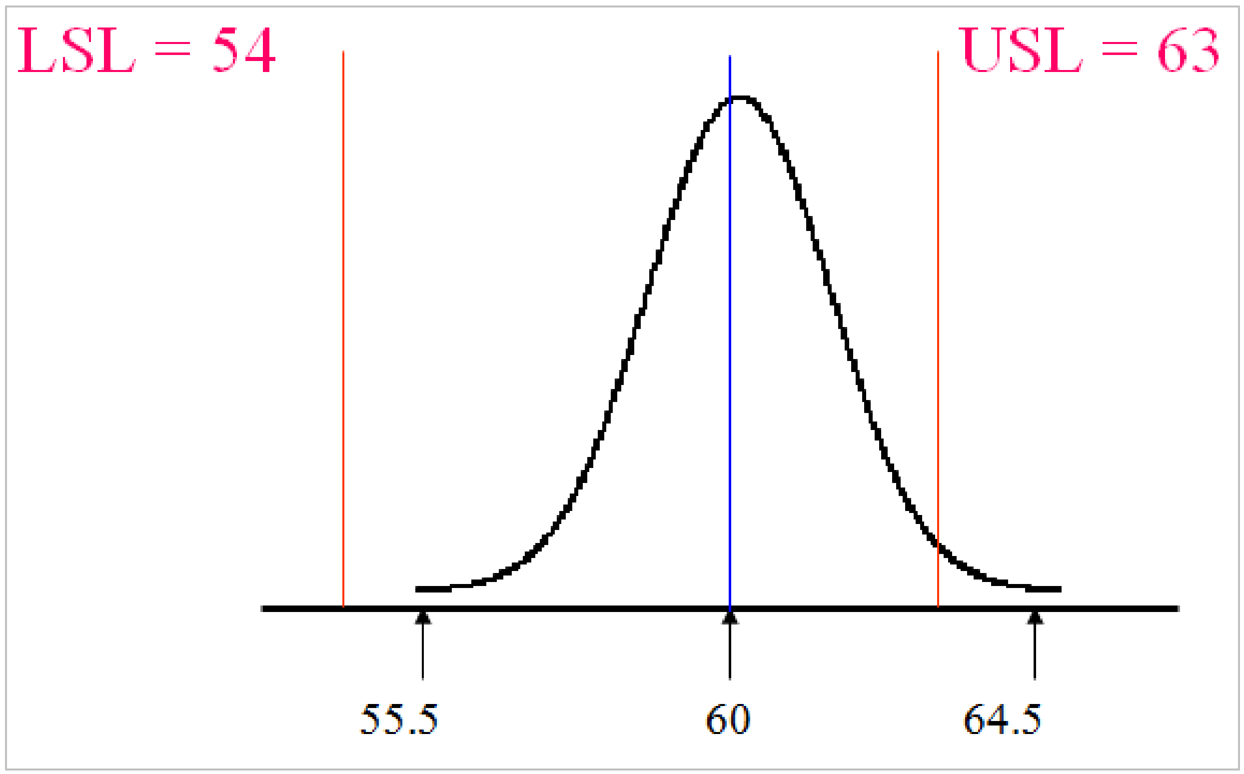
Process Capability Indices

$$C_{PU} = \frac{USL - \mu}{3\sigma_{ST}}; \quad C_{PL} = \frac{\mu - LSL}{3\sigma_{ST}}$$

$$C_{PK} = \min\{C_{PU}, C_{PL}\}$$

Find C_{PK} for this process.



Process Capability Indices

Below is the solution to the C_{pk} calculation from the previous slide.

Note that the natural limits represent $6\sigma_{ST}$ - this is used to calculate σ_{ST} from the information provided on the slide.

$$\sigma_{ST} = \frac{64.5 - 55.5}{6} = \frac{9}{6} = 1.5$$

$$C_{PU} = \frac{63 - 60}{3(1.5)} = 0.67; \quad C_{PL} = \frac{60 - 54}{3(1.5)} = 1.33$$

$$C_{pk} = 0.67$$

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Process Capability Indices

Capability indices measure the number of normal distributions that can be placed between the specification limits.

Assuming a normal distribution for the process characteristic, 99.73% of all possible process values will lie in a $\pm 3\sigma$ window of the process mean μ (a 6σ process window).

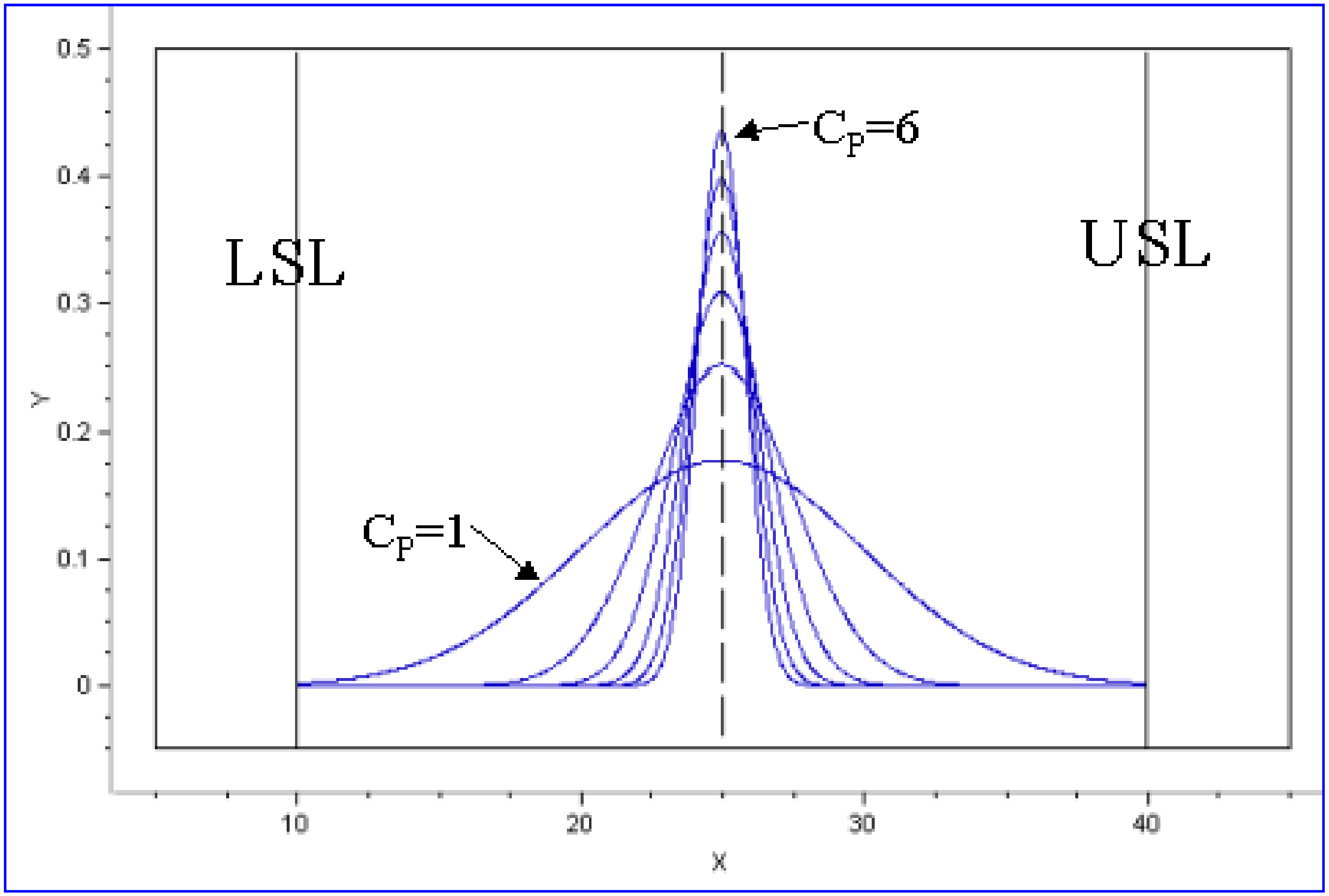
The capability indices address the issue of how many of these 6σ windows fit within the total specification range for the characteristic of interest.

C_P assumes that the process is centered at the target and divides the specification range by 6σ.

C_{PK} picks the **shortest interval from the process mean to one of the specification limits**, and divides this interval by 3σ.

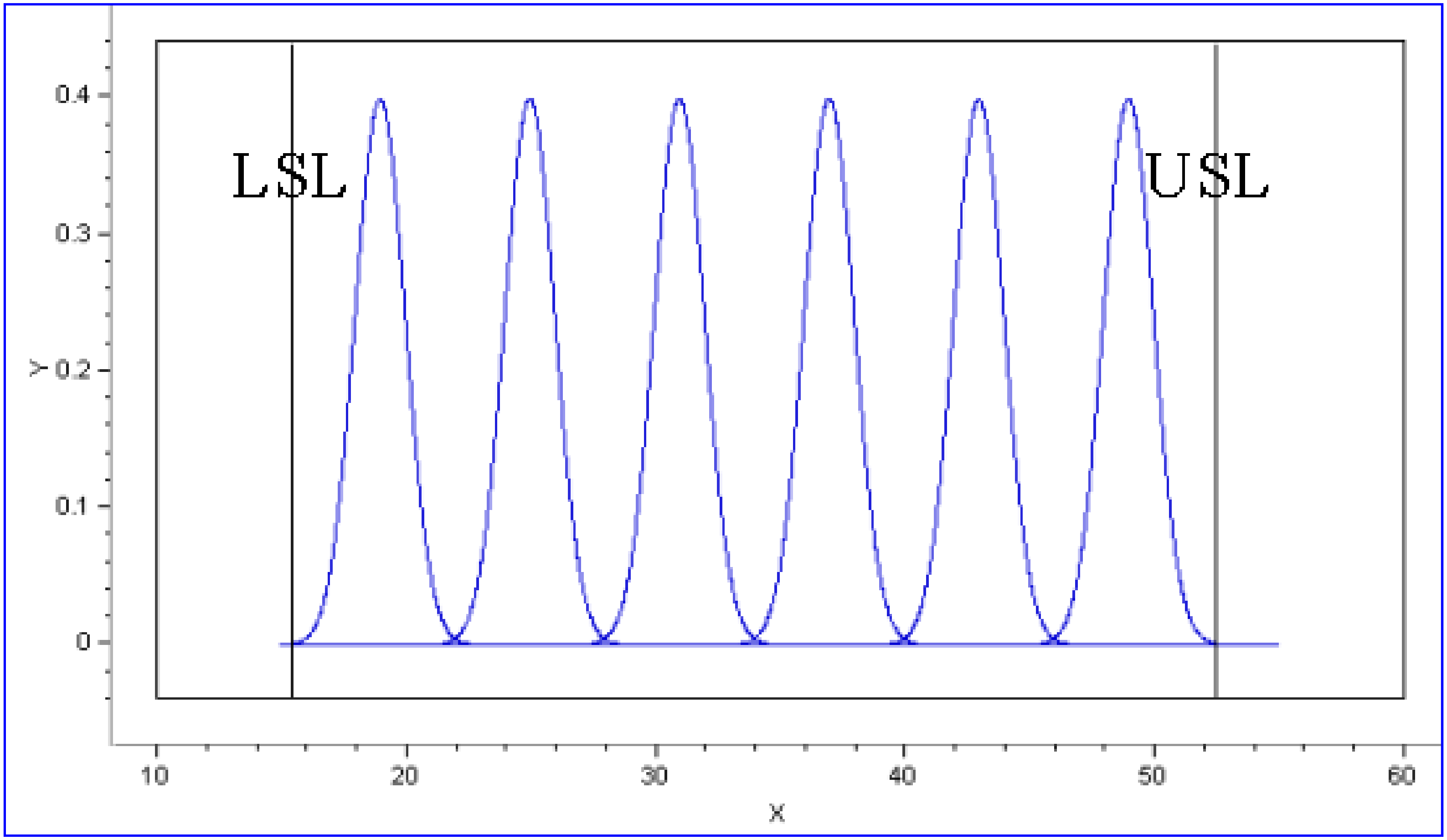
Process Capability Indices

This figure relates the spread of normal distributions within the specification limits to C_P values:



Process Capability Indices

A C_P of 6 indicates that six normal distributions fit within the specification limits:



Interpretation of Indices

What are good values of C_P and C_{PK}?

There are many guidelines for acceptable values depending on the industry group, the specific company, the specific division of the company, or specific quality standards (e.g., Ford Q1).

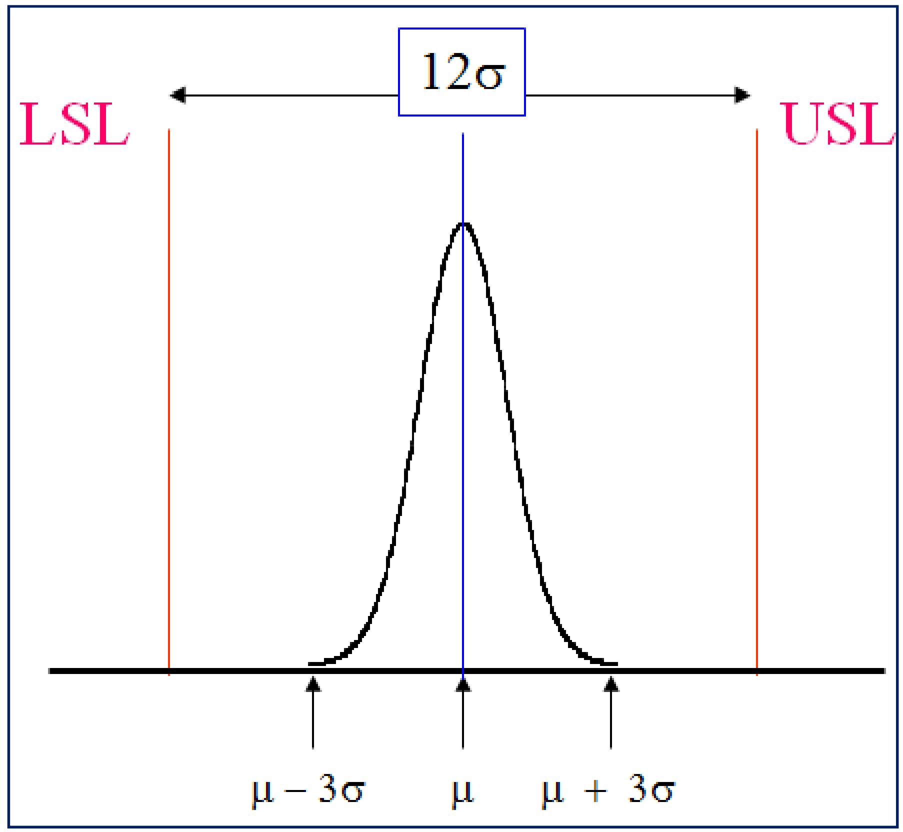
Common guidelines:

- **Joseph Juran** proposed that a C_{PK} should be **at least 1.33**.
- Motorola, as part of their Six Sigma implementation, proposed an acceptable C_P of at least 2.0 and C_{PK} of at least 1.5.
- QS9000 requires at least 1.67 for C_{PK}.
- A barely capable process is considered to have a C_{PK} =1.0.

Interpretation of Indices

Motorola's recommended values stemmed from equating a $C_P = 2$ to a centered process where the upper and lower specification limits were each 6σ from the process target.

A quality level of 6σ equates to a ppm rate of 0.002.

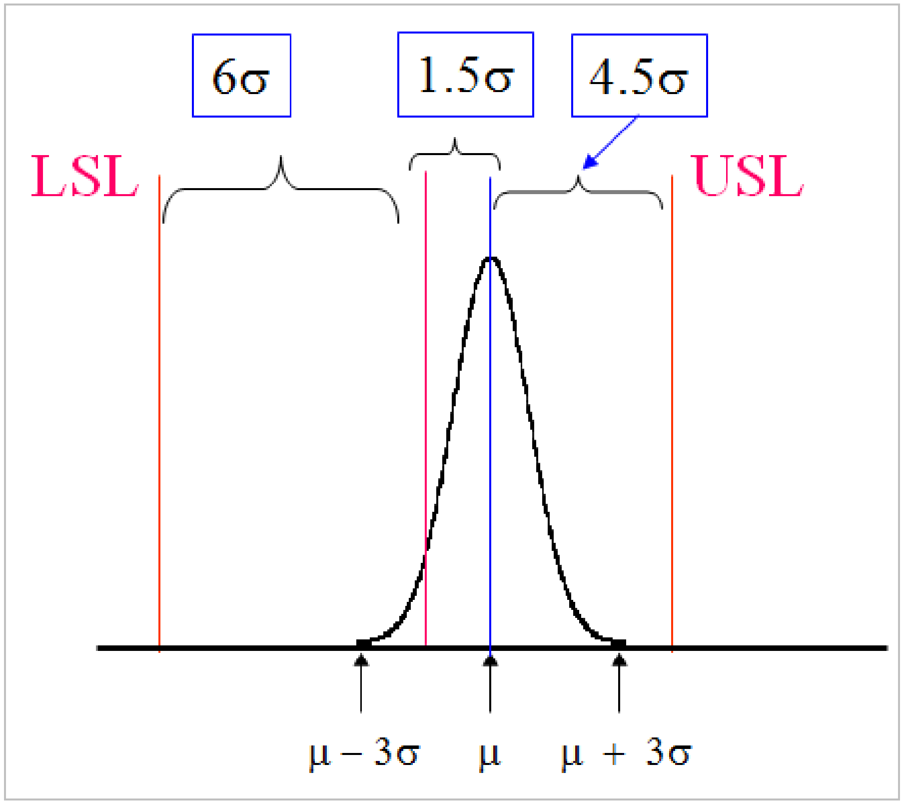


Interpretation of Indices

Then, Motorola built in the idea that the process might vary in the long term by as much as 1.5σ.

Since a C_{PK} = 1.5 gives a defect rate of 3.4 ppm, they deemed this acceptable.

This corresponds to a Sigma Level of 4.5, or 3.4 ppm, which is considered the Six Sigma definition of world class performance.



Long Term vs. Short Term Variation

The Automotive Industry Action Group (AIAG) developed a distinction between what they deemed short term and long term variation, and the associated capability.

C_{pk} is calculated based upon common cause variation, often referred to as short term variation – rational subgroups are used to estimate σ.

Process capability indices based upon long term variation are referred to as P_p and P_{pk}.

The indices are calculated in the same manner as C_p and C_{pk} except that a long term estimate of σ is used – usually the standard deviation of the entire data set.

Unfortunately, for an in control process there is little distinction between long term and short term capability.

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Long Term vs. Short Term Variation

If a process is in a state of statistical control, then only common cause variation is present over time.

The total amount of variation in the process is predictable.

Control charts work because their limits bound the amount of common cause variation present in the process.

If a process is not in a state of statistical control (unstable), then common cause and special cause variation are both present.

Special cause variation is unpredictable, therefore the total amount of variation over time is also unpredictable.

Process capability cannot be predicted for an unstable process, and long term capability estimates amount to artifacts of the sample data used – they do not predict future process capability.

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Long Term vs. Short Term Variation

If a process is in a state of statistical control, then any perceived differences between long and short term estimates of the process standard deviation σ are again mere artifacts of the samples.

Theoretically, for an in control process, the short term and long term standard deviations are the same.

Hence, for an in control process there is no actual difference between long and short term capability.

For an unstable or out of control process, long term capability is meaningless, since total variation cannot be predicted over time.

We strongly suggest that P_p and P_{pk} not be used for these reasons.

The Problems of Cp and Cpk

Although process capability indices appear conceptually to be a nice way to summarize the capability of a process, they are not without significant drawbacks.

Unfortunately, too many practitioners use C_p and C_{pk} naively and are completely oblivious to significant technical and conceptual problems.

One conceptual issue with C_{pk} is that the underlying information about the process mean and the process standard deviation are confounded, because they form a ratio in the calculation.

Another issue is that the underlying information about the mean of the process, the stability, and the variation are lost in the calculation of the index – never try to interpret C_{pk} without associated control charts.

The Problems of Cp and Cpk

Much more serious issues exist for C_p and C_{pk}.

We will focus on only C_{pk} for the remainder of the talk.

- Since the true process mean μ and true process standard deviation σ are unknown, they must be estimated from sample.

- Once the sample estimates are inserted into the calculation, the estimate, $\hat{C}_{pk}$, is a random variable or statistic.

  It has an associated probability distribution that describes its random behavior.

- Regrettably, too many practitioners view the estimate $\hat{C}_{pk}$ deterministically – they view it as the true capability, when in fact the true capability is usually quite different than the estimated value.

The Problems of Cp and Cpk

As an estimate, $\hat{C}_{pk}$ has a number of very undesirable properties. Among them are:

- The probability distribution describing its random behavior is complicated and mathematically intractable.

  Researchers over the years have developed useful approximations – even they are complicated.

- In smaller samples (say less than 50), estimated capability is biased high (over estimates the true capability).

  In larger samples, the estimated capability is biased low (under estimates the true capability).

- The amount of bias increases the more off target the true process performance.

The Problems of Cp and Cpk

- In small samples, the estimates of capability are highly unstable and vary wildly from sample to sample.

  They are very poor estimates of the true capability.

- The higher the true capability the more variable the sample estimates.

  Ironically, the more capable your process, the harder it becomes to estimate the capability using $\hat{C}_{pk}$.

- Estimating the true capability to a second decimal place requires sample sizes greater than 500 (the actual sample size depends upon the true capability) – of course the estimate is now biased low, but less variable.

We now illustrate the variability of $\hat{C}_{pk}$ using a virtual Quincunx.

The Problems of Cp and Cpk

To further illustrate the undesirable behavior of $\hat{C}_{pk}$ we show the results of a simulation.

We begin by using a sample size of n = 5 to generate the sample estimates. There are 5,000 repetitions of the simulation.

For the simulation we use the following calculations. The true C_{pk} in the simulation is 1.5.

$$C_{pl} = \frac{\mu - LSL}{3\sigma} = \frac{15.5 - 11}{3(1)} = 1.5, \quad C_{pu} = \frac{USL - \mu}{3\sigma} = \frac{20 - 15.5}{3(1)} = 1.5$$

$$C_{pk} = Min\left\{C_{pl}, C_{pu}\right\} = 1.5$$

$$\hat{C}_{pl} = \frac{\bar{X} - 11}{3S}, \quad \hat{C}_{pu} = \frac{20 - \bar{X}}{3S}$$

$$\hat{C}_{pk} = Min\left\{\hat{C}_{pl}, \hat{C}_{pu}\right\}$$

MS1

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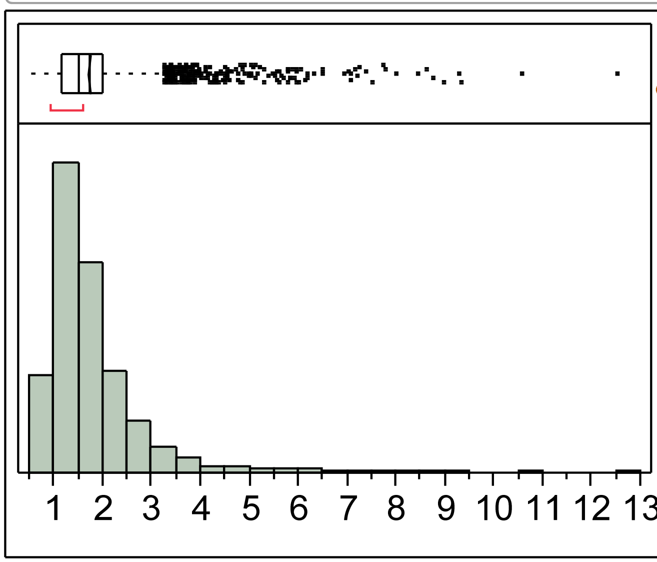
The Problems of Cp and Cpk

Below are the results of the simulation. The range of $\hat{C}_{pk}$ values is from 0.565 to 12.52 for the same stable process.

Also notice that the mean estimated C_{pk} value is 1.728. As expected we have overestimated the true C_{pk} in a small sample.

Distributions

Cpk



Quantiles

100.0%	maximum	12.521
99.5%		6.176
97.5%		3.900
90.0%		2.694
75.0%	quartile	2.004
50.0%	median	1.511
25.0%	quartile	1.177
10.0%		0.977
2.5%		0.814
0.5%		0.714
0.0%	minimum	0.565

Moments

Mean	1.7281641
Std Dev	0.8863106
Std Err Mean	0.0125356
Upper 95% Mean	1.7527394
Lower 95% Mean	1.7035889
N	4999

Slide 33

MS1 Phil - why is N 4999?

Mia Stephens, 1/13/2009

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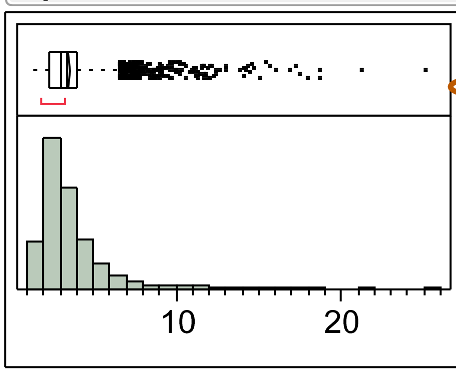
The Problems of Cp and Cpk

Let's redo the simulation, but in this case we use a true underlying C_{pk} = 3.0. We expect that the variation in $\hat{C}_{pk}$ values will approximately double.

The range of estimates is from 1.13 to 25.043. The mean estimate is 3.46, so again we overestimate the true capability.

Distributions

Cpk



Quantiles

100.0%	maximum	25.043
99.5%		12.351
97.5%		7.800
90.0%		5.389
75.0%	quartile	4.009
50.0%	median	3.021
25.0%	quartile	2.354
10.0%		1.954
2.5%		1.628
0.5%		1.428
0.0%	minimum	1.130

Moments

Mean	3.4563282
Std Dev	1.7726213
Std Err Mean	0.0250712
Upper 95% Mean	3.5054787
Lower 95% Mean	3.4071778
N	4999

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The Problems of Cp and Cpk

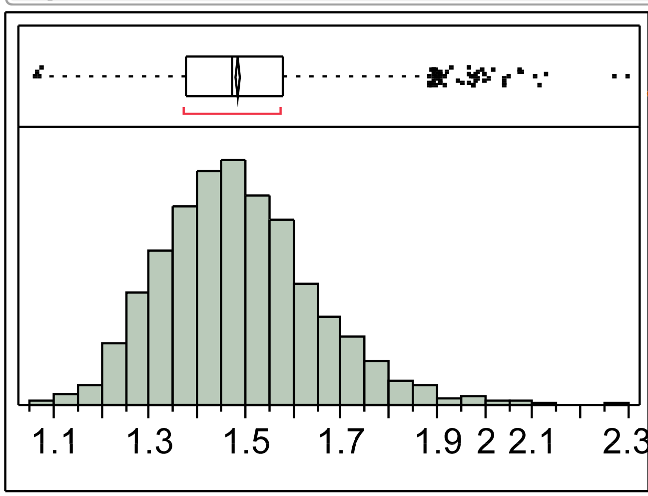
Suppose we increase the sample size to n = 50 in our simulation. Again the true $C_{pk} = 1.5$.

Notice that the range of estimates is from 1.058 to 2.294.

This time the mean estimate is 1.48 is slightly biased below 1.5.

Distributions

Cpk



Quantiles

100.0%	maximum	2.2943
99.5%		1.9722
97.5%		1.8217
90.0%		1.6896
75.0%	quartile	1.5777
50.0%	median	1.4717
25.0%	quartile	1.3752
10.0%		1.2921
2.5%		1.2151
0.5%		1.1389
0.0%	minimum	1.0581

Moments

Mean	1.483855
Std Dev	0.1559219
Std Err Mean	0.0022051
Upper 95% Mean	1.4881779
Lower 95% Mean	1.4795321
N	5000

The Problems of Cp and Cpk

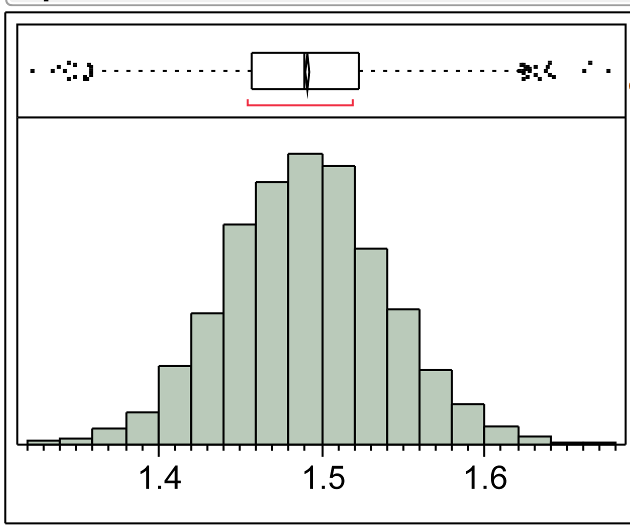
Finally, we increase the sample size to n = 500 in our simulation. Again the true $C_{pk} = 1.5$.

The range of estimates is from 1.32 to 1.675.

Even with n = 500 we cannot nearly specify C_{pk} to one decimal place.

Distributions

Cpk



Quantiles

100.0%	maximum	1.6753
99.5%		1.6189
97.5%		1.5876
90.0%		1.5523
75.0%	quartile	1.5227
50.0%	median	1.4901
25.0%	quartile	1.4572
10.0%		1.4290
2.5%		1.3969
0.5%		1.3683
0.0%	minimum	1.3225

Moments

Mean	1.4904413
Std Dev	0.0484471
Std Err Mean	0.0006851
Upper 95% Mean	1.4917845
Lower 95% Mean	1.4890981
N	5000

The Problems of Cp and Cpk

Perhaps the greatest misuse of C_{pk} is to view individual estimates deterministically.

As the simulations illustrate, C_{pk} is a poor metric to drive continuous improvement, since the estimates are so variable.

It is virtually impossible to discern true improvements using C_{pk}.

We strongly argue against C_{pk} as a Key Performance Indicator for continuous improvement projects or intitatives.

Far too much time and effort are wasted by companies chasing phantom process problems based upon the latest C_{pk} estimates.

Such resources would be far better spent on real continuous improvement projects and customer service rather than endless C_{pk} engineering efforts.

The Problems of Cp and Cpk

We illustrate the misuse of C_{pk} as a performance metric with a case study.

A supplier of photoresist (used to imprint circuit images on silicon wafers) for a semiconductor manufacturer is required to maintain a minimum C_{pk} of 1.5 for the photospeed of the resist.

The supplier ships around 25 batches per month and is required to report an estimated C_{pk} value each month.

In months where C_{pk} falls below 1.5, the company is required to look for assignable causes and file a corrective action report with the customer's Quality Assurance Department.

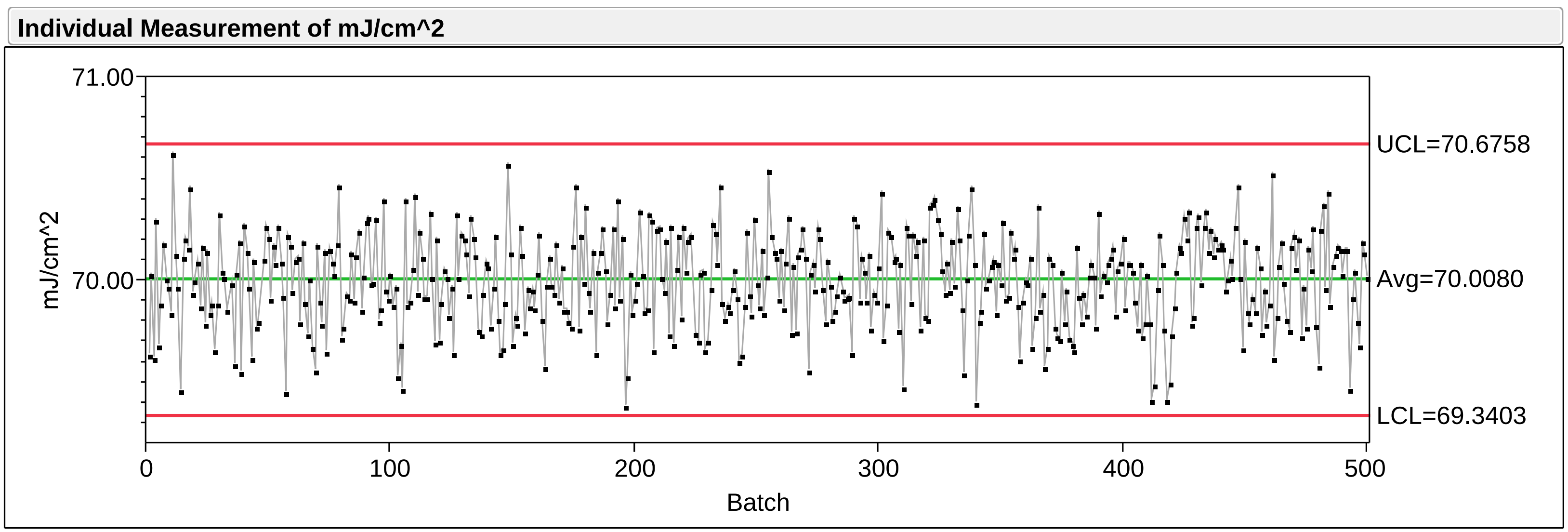
In the entire manufacturing history with this customer no assignable causes have been identified in the months where C_{pk} appears low.

The Problems of Cp and Cpk

Over this time period, no customer complaints have been received from the fabrication facilities actually using the photoresist.

Below is a control chart for 500 batches shipped over the last 20 months. Photospeed is measured in mJ/cm^2 (millijoules).

The process appears to be in statistical control - the variation from batch to batch is common cause.



The Problems of Cp and Cpk

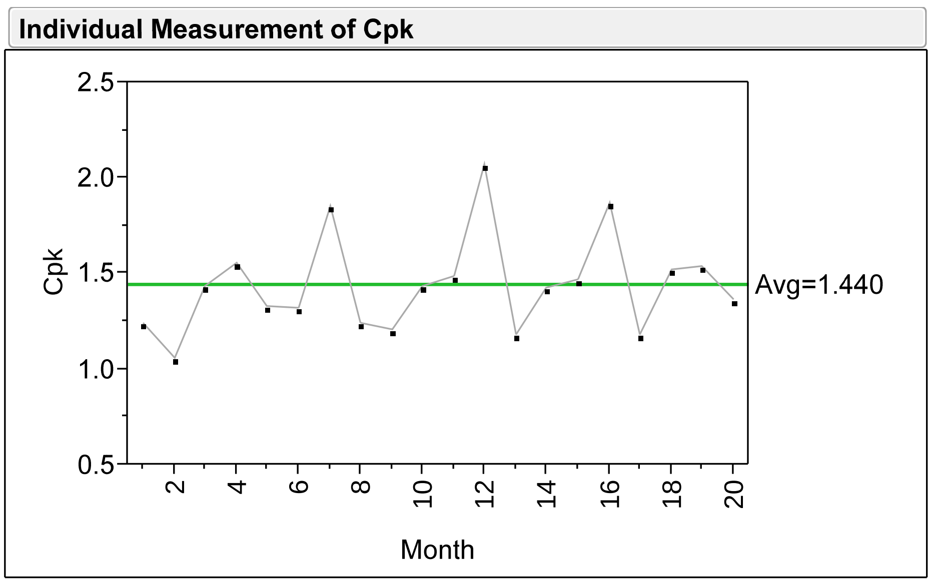
Below is a run chart of the 20 monthly estimated C_{pk} values.

The process is in control, so no assignable causes of variation exist.

Note the average C_{pk} estimate is very close to the goal of 1.5

Ten of the values are below 1.5, causing the supplier to waste time looking for nonexistent assignable causes of variation.

The specifications are 70 ± 1.0 mJ



Confidence Intervals for Cpk

Statisticians often quantify the uncertainty in an estimate of a parameter value, in our case C_{pk}, through the use of confidence intervals.

The margin of error values provided with polling data form a confidence interval for the true, but unknown, proportion of voters favoring a candidate.

As an example 45% of voters favored candidate X with a margin of error of 3% meaning the true proportion of voters favoring candidate X is somewhere between 42% and 48%.

Similarly, confidence intervals for the true C_{pk} can be formed from sample C_{pk} estimates.

We omit the complicated math required to calculate these intervals.

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Confidence Intervals for Cpk

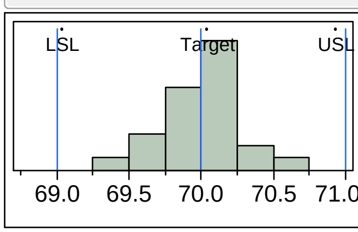
The JMP statistical software computes C_{pk} estimates with confidence intervals.

Below is the capability report for month 1 of the photospeed data.

The confidence interval for C_{pk} is 0.868 to 1.618 - we are confident the true C_{pk} is somewhere in this interval.

Distributions Month=1

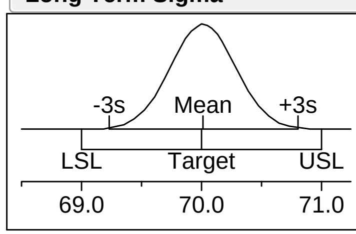
mJ/cm^2



Capability Analysis

Specification	Value	Portion	% Actual
Lower Spec Limit	69	Below LSL	0.0000
Spec Target	70	Above USL	0.0000
Upper Spec Limit	71	Total Outside	0.0000

Long Term Sigma



Sigma = 0.26304

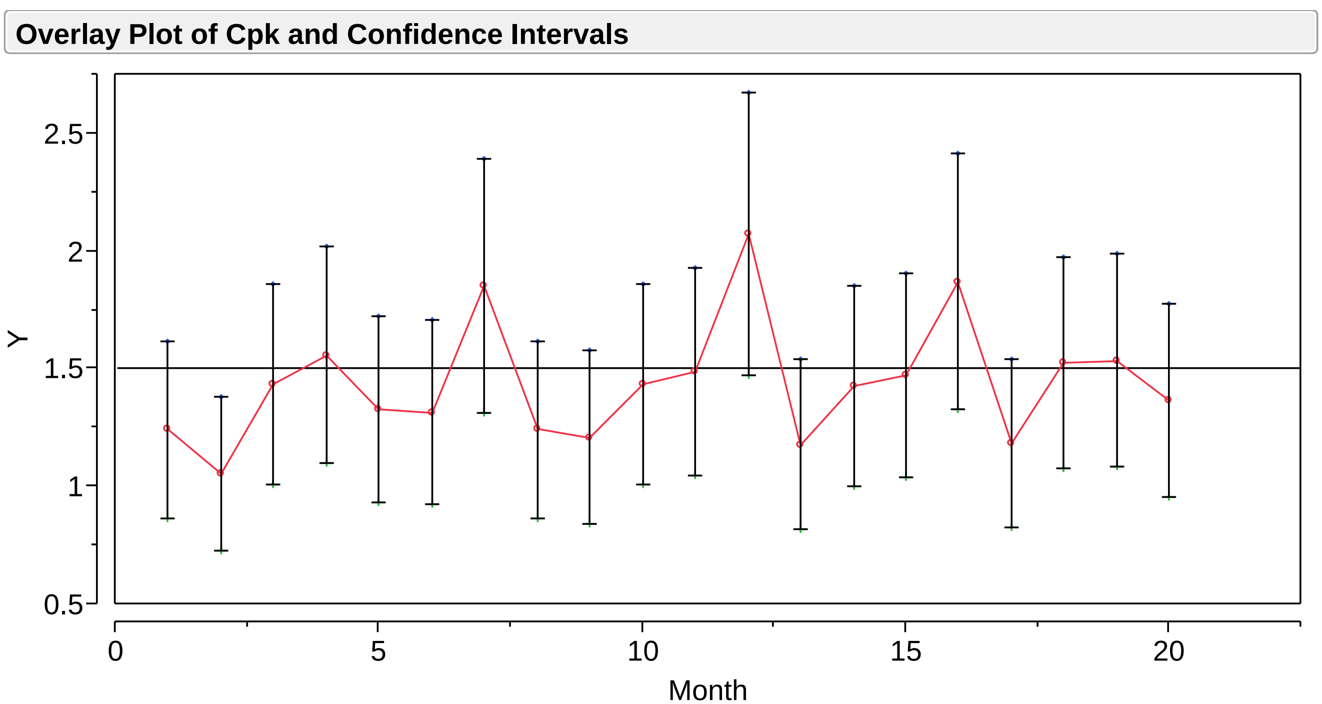
Capability	Index	Lower CI	Upper CI
CP	1.267	0.911	1.623
CPK	1.243	0.868	1.618
CPM	1.264	0.916	1.611
CPL	1.292	0.904	1.675
CPU	1.243	0.868	1.614

Portion	Percent	PPM	Sigma Quality
Below LSL	0.0053	53.3929	5.375
Above USL	0.0096	96.1715	5.229
Total Outside	0.0150	149.5644	5.116

Confidence Intervals for Cpk

Below is a plot of the 20 C_{pk} estimates, and the associated confidence intervals. The intervals are only approximate for C_{pk}.

All of the intervals overlap and contain 1.5 as a possible true value (except for Month 2, but it is close).



Cpk for Non-normal Distributions

Process capability indices such as C_{pk} only work correctly if the distribution of the process measurement is normal.

This is a strong assumption.

For non-normal distributions, C_{pk} will not correctly predict process performance.

For skewed distributions, the estimates of process capability are particularly bad.

In fact, for skewed distributions it is not clear that C_{pk} should be used at all, since each tail of the distribution should be considered separately with regard to the specification limits.

In any case, various attempts have been made to apply C_{pk} to non-normal data, and all of the approaches have drawbacks.

Cpk for Non-normal Distributions

Three basic approaches exist to estimate C_{pk} for non-normal data:

1) Estimate C_{pk} as if the data are normally distributed;
2) Use a nonlinear transformation (e.g, Box Cox) to create a more normal looking distribution;
3) Mimic C_{pk} using percentiles instead of moments of the distribution.

The first approach is completely wrong and will result in very inaccurate capability estimates.

The second approach is most commonly recommended, but has substantial technical flaws that are often not understood.

The third approach is now considered to be best practice.

Cpk for Non-normal Distributions

Transformations to normality often do not exist.

Many non-normal distributions simply cannot be transformed to a scale where they appear normal-like.

The nonlinear transformation loses the relationship between the original specification limits (including target) and the C_{pk} estimate on the transformed scale – this is due to what statisticians call inverse transformation bias.

The estimated C_{pk} only applies directly to the transformed scale and not directly to the original scale.

Also, some transformation methods, such as the Pearson transformation, require the estimates of distribution parameters that are themselves wildly unstable.

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Cpk for Non-normal Distributions

We illustrate the percentile method, but omit the mathematical details.

The JMP software (and a few other packages) implements this approach for non-normal C_{pk} estimates of capability.

We note that one drawback of the percentile method is that confidence intervals are not possible to compute.

Case Study: In another photoresist example, the manufacturer of the product has a specification of 16.5 mJ ± 3.5 mJ.

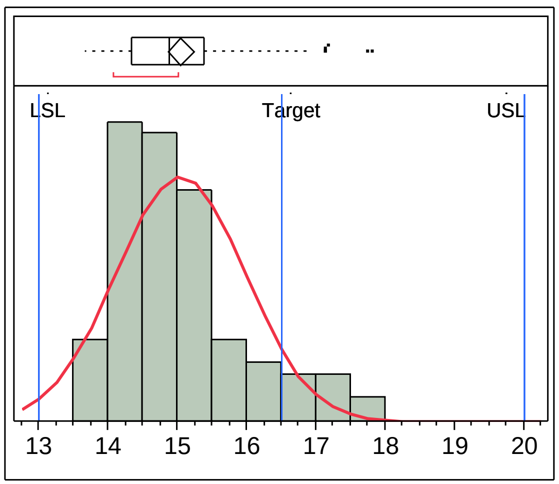
Based upon the last 100 batches, a C_{pk} value is estimated.

From the histogram on the next slide we see that the distribution has a long right tail – it is skewed right.

The engineers discovered that no skewed distribution fit the data correctly, and no transformation made it normal-like in appearance.

Cpk for Non-normal Distributions

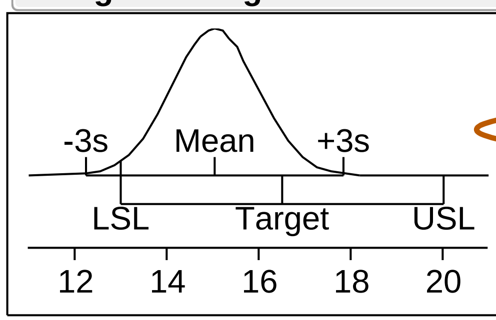
Case Study cont'd: The histogram of the distribution and the C_{pk} estimate of 0.727 assuming a normal distribution –this is not an appropriate estimate. A normal curve is overlaid on the histogram.



Capability Analysis

Specification	Value	Portion	% Actual
Lower Spec Limit	13	Below LSL	0.0000
Spec Target	16.5	Above USL	0.0000
Upper Spec Limit	20	Total Outside	0.0000

Long Term Sigma



Sigma = 0.93502

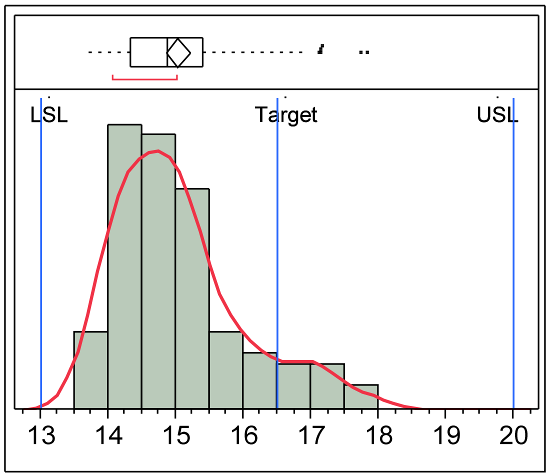
Capability	Index	Lower CI	Upper CI
CP	1.248	1.074	1.421
CPK	0.727	0.607	0.848
CPM	0.673	0.595	0.751
CPL	0.727	0.606	0.847
CPU	1.768	1.513	2.022

Portion	Percent	PPM	Sigma Quality
Below LSL	1.4565	14565.082	3.682
Above USL	0.0000	0.0564	6.805
Total Outside	1.4565	14565.138	3.682

About 1.5 % of batches are predicted to be out of spec.

Cpk for Non-normal Distributions

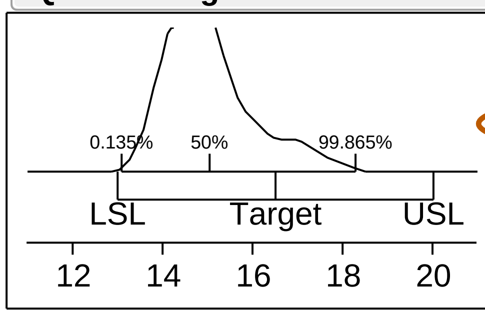
Case Study cont'd: The histogram of the distribution with the nonparametric density overlaid, and the C_{pk} estimate of 1.052 based on the percentile method.



Capability Analysis

Specification	Value	Portion	% Actual
Lower Spec Limit	13	Below LSL	0.0000
Spec Target	16.5	Above USL	0.0000
Upper Spec Limit	20	Total Outside	0.0000

Quantile Sigma



Capability	Index
CP	1.349
CPK	1.052
CPM	0.553
CPL	1.052
CPU	1.503

Portion	Percent	PPM	Sigma Quality
Below LSL	0.0654	654.4174	4.714
Above USL	0.0000	0.0000	8.652
Total Outside	0.0654	654.4174	4.714

About 0.07% of batches are predicted to be out of spec.

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Cpk for Batch Control

Although it is not obvious, C_{pk} does not imply sufficiently low batch to batch variability in a product for customer use.

There is nothing in the calculation that explicitly considers batch to batch variation.

Because C_{pk} confounds the process mean and the process standard deviation, it is entirely possible to realize significant variation between batches while maintaining an acceptable C_{pk}.

Batch-to-batch variation can be monitored directly and controlled with a Moving Range control chart.

Other approaches have been suggested over the years, but C_{pk} is not a good option for this case.

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Cpk for Multiple Characteristics

Many processes have numerous characteristics for which capability is assessed.

A C_{pk} value is estimated for each characteristic.

Unfortunately, there is no mathematically tractable way to combine C_{pk} estimates into one overall estimate that is meaningful or interpretable.

Strong correlations often exist among the numerous characteristics, making it impossible to view the C_{pk} estimates individually.

As an example, for a coating process, two important characteristics are coating viscosity and coated thickness.

Unfortunately the two characteristics are negatively correlated and cannot be interpreted individually.

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Cpk for Multiple Characteristics

In these cases, one could still look at the numerous characteristics in terms of the means and standard deviations, and take correlation into account in the interpretation and subsequent actions.

As an example, the JMP 8 statistical software implements a multivariate plot that can be very helpful in graphically spotting process capability issues with multiple responses.

The graphical display is referred to as a **goal post plot**.

We will illustrate the plot with a semiconductor case study.

We have a semiconductor manufacturing process with 128 measured quality characteristics.

There are specification limits for each of the characteristics, and we have capability data based upon 1455 lots.

Cpk for Multiple Characteristics

Many of the characteristics are highly correlated, so one has to be careful in deciding to try improve performance in one characteristic where it might degrade the performance of one or more other characteristics.

The idea of the plot is to graph the means and standard deviations of each characteristic, normalized by the associate specification limits.

The next slide displays the goal post plot.

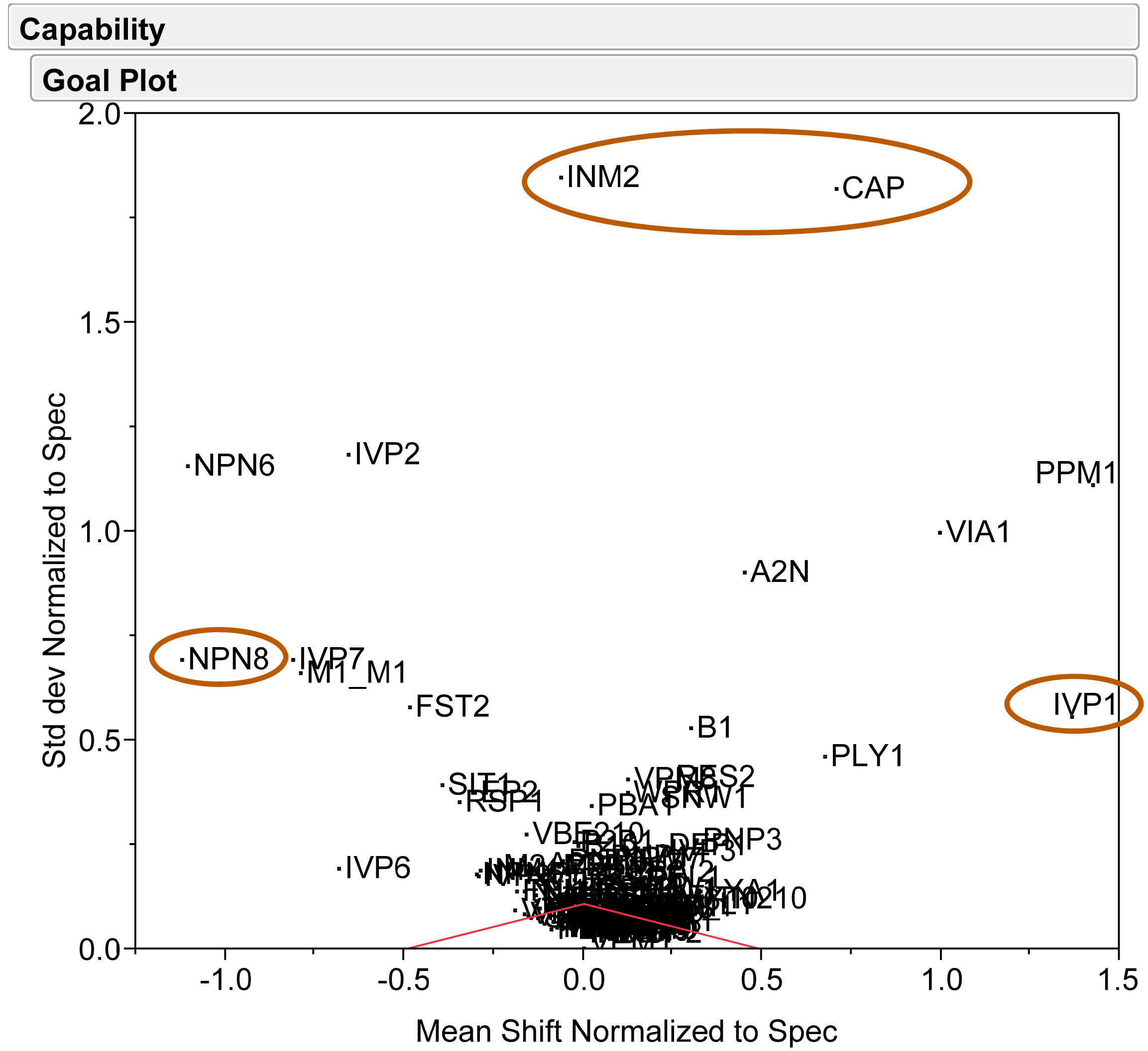
The triangle at the bottom of graph encloses characteristics with a C_{pk} of 1.5 or better.

Cpk for Multiple Characteristics

INM2 and CAP are close to target, but have large standard deviations, causing low capability.

NPN8 and IVP1 have lower standard deviations but are way off target, causing low capability.

The plot is more informative than just C_{pk} values.



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Alternatives to Cpk

In general no simple alternative to C_{pk} exists, nor should one exist.

We agree with Peter Nelson, who writes:

"…it is clear from a statistical perspective that the concept of attempting to characterize a process with a single number is fundamentally flawed." (1992, JQT, ASQ)

A better approach to capability assessment is through the statistical concept of **tolerance intervals**.

Tolerance intervals were first suggested as a method of capability assessment in the 1950's, however, over time, virtually no effort has been made to do so.

The lack of interest most likely stems from a lack of knowledge of tolerance intervals and how to interpret them.

Alternatives to Cpk

A tolerance interval is a statistical interval that is calculated from sample data, and places a lower bound on the proportion of the population that falls within that interval.

Using tolerance limits, one can confidently tell the customer the maximum variation in product quality that will be realized over time – this is far more useful information than supplying a single C_{pk} estimate.

By aligning the specification limits with the tolerance limits, one can also achieve an upper bound on the total amount of product that will fall outside of the specification limits.

It was originally thought that C_{pk} provided such an upper bound.

Unfortunately, due to their inherent variability, the C_{pk} estimates do not provide such a bound.

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Alternatives to Cpk

A tolerance interval is specified in terms of the proportion of the population it bounds, usually 0.90 or 0.95, and a confidence that the interval actually captures that proportion, usually 95%.

Tolerance intervals are either based on the assumption of a normal distribution (a hard assumption) or a nonparametric version exists for non-normal data.

In order for tolerance intervals to predict process capability, we still must have an in control process.

A stable process is a prerequisite to predicting process performance.

We omit the mathematical details of tolerance intervals and illustrate the concept with an example.

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Alternatives to Cpk

Case Study: A manufacturer provides steel clips to an automobile manufacturer.

The clips are used in a door assembly operation.

The clip gap has a specification range of 14.4 mm to 15.8 mm.

SPC is maintained for the clip manufacturing line by taking a daily subgroup and measuring the gap.

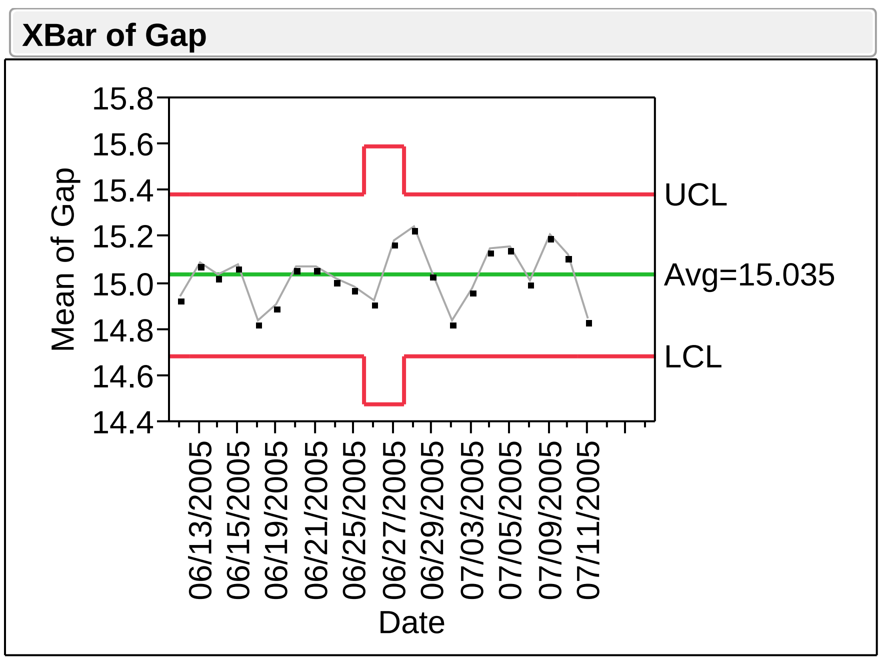
The data for the past 70 days production is being used to estimate process capability.

Using the JMP 8 statistical software we will perform a process capability assessment using C_{pk} and a tolerance intervals.

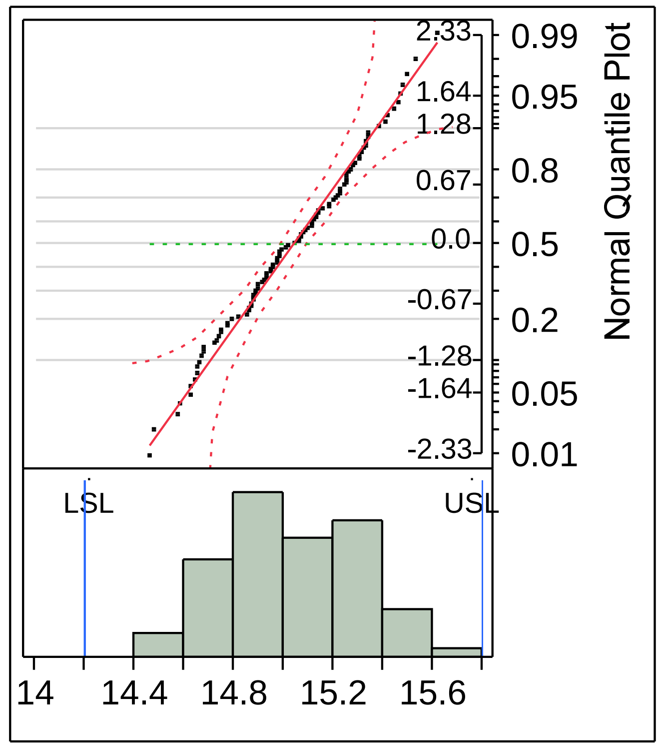
Alternatives to Cpk

Case Study: From the XBar chart, the process appears stable or in control.

From the Normal Quantile Plot, the data appear to come from a normal distribution – plots along a straight line.



Note: The sigma was calculated using the range.



Alternatives to Cpk

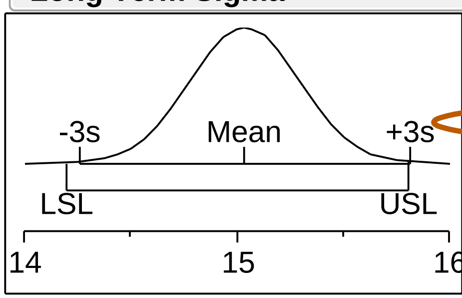
Case Study: The usual normal based C_{pk} estimate is 0.987 with a confidence interval of 0.84 to 1.14.

Based on the C_{pk} estimate about 0.214% of the clips will be out of specification.

Capability Analysis

Specification	Value	Portion	% Actual
Lower Spec Limit	14.2	Below LSL	0.0000
Spec Target	.	Above USL	0.0000
Upper Spec Limit	15.8	Total Outside	0.0000

Long Term Sigma



Sigma = 0.25824

Capability	Index	Lower CI	Upper CI
CP	1.033	0.892	1.173
CPK	0.987	0.838	1.136
CPM	.	.	.
CPL	1.078	0.917	1.238
CPU	0.987	0.837	1.135

Portion	Percent	PPM	Sigma Quality
Below LSL	0.0608	608.4592	4.735
Above USL	0.1534	1533.9917	4.461
Total Outside	0.2142	2142.4509	4.356

Alternatives to Cpk

Case Study: A tolerance interval is calculated to bound 95% of the entire population of clip gaps for the process with 95% confidence.

The interval estimates that at least 95% of all manufactured clips have a gap of 14.46 to 15.61.

Tolerance Intervals

Proportion	Lower TI	Upper TI	1-Alpha
0.950	14.46014	15.61063	0.950

Notice that a tolerance interval on 99% of the population of clip gaps is nearly equal to the spec limits.

Tolerance Intervals

Proportion	Lower TI	Upper TI	1-Alpha
0.990	14.27945	15.79132	0.950

Summary

C_{pk} continues to be used as a measure of process capability, with near religious ferocity.

However, the usefulness of C_{PK} as an indicator of true process behavior is questionable, given:

- The **strong assumption of normality** for the process distribution,
- The **highly unstable behavior of C_{PK} estimates**, even in moderate sample sizes, and
- The effect of **lack of independence** on estimates.

Summary

C_{pk} may have some usefulness for high volume, discrete parts manufacturing.

But, in general, C_{pk} is not useful as a basis for either process improvement or troubleshooting.

The continued emphasis on C_{pk} wastes valuable resources spent chasing nonexistent or vague process problems – it can lead to process tampering and actually lessen capability.

Furthermore, reliance on C_{pk} as a measure of process performance can mask opportunities for real improvement.

It is our opinion that continued reliance and emphasis on C_{pk} has very real negative economic consequences for industry, and its use should be greatly diminished or discontinued all together.

Summary

The simplicity of interpretation of C_{pk} no doubt drives the widespread use and abuse.

However, the simplicity of interpretation belies the substantial technical problems with C_{pk} estimates of process capability, as we have discussed.

Over the past 20 years numerous published articles and several textbooks have well documented the shortcomings of C_{pk}.

An excellent (and long) article thoroughly discussing C_{pk} is found in the January 2002 issue of the Journal of Quality Technology, ASQ. The article is titled "*Process Capability Indices - A Review, 1992-2000 (With Subsequent Discussions and Response).*" The primary authors are S. Kotz and N. Johnson. Note that there are many discussants to the main article.